Appendix of the paper "A comparison of priority rules for the job shop scheduling problem under different flow time- and tardiness-related objective functions"

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In this appendix, we give more information on the priority rules used in the paper *"A comparison of priority rules for the job shop scheduling problem under different flow time- and tardiness-related objective functions"*. In Section 1, the abbreviations used are outlined. Section 2 list the single priority rules together with their mathematical formulation. For this overview, we based ourselves on the papers of, among others, Panwalkar and Iskander (1977), Blackstone et al. (1982), Baker (1984), Russell et al. (1987), Vepsalainen and Morton (1987), Anderson and Nyirenda (1990), Waikar et al. (1995), Holthaus and Rajendran (1997), Rajendran and Holthaus (1999), Jayamohan and Rajendran (2000), Holthaus and Rajendran (2000), Dominic et al. (2004), Mizrak and Bayhan (2006) and Chiang and Fu (2007). Finally, in Section 3, more information is given on the combination of the single rules.

1 Abbreviations

In order to explain the abbreviations used, an example of a disjunctive graph for a JSSP with three machines and three jobs is given in figure 1. In this example, each job j (j = 1, 2 or 3) has three operations to be performed on one of the machines i (i = 1, 2 or 3), which is denoted as O_{ij}. The index q is used to refer to the q^{th} operation of a job. For example, operation O_{22} is the first operation of job 2 on machine 2, indicated by j = 2, i = 2 and q = 1.

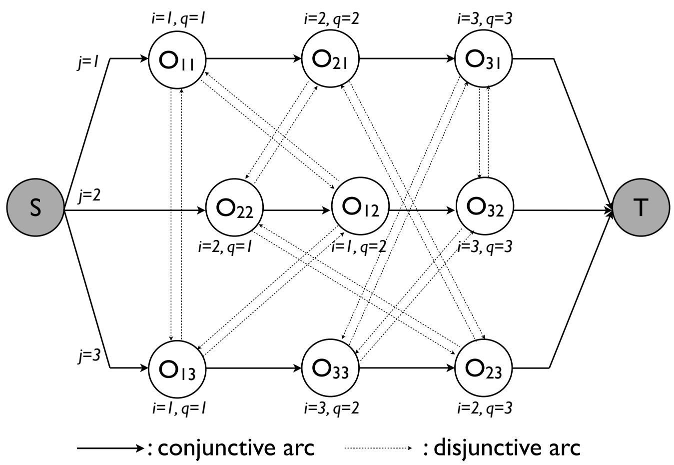

Figure 1: Disjunctive graph representation of a 3×3 job shop

In section 2, we calculate the priority value (Z_j) for each job j at time τ. We assume that this corresponds with the current operation q to be performed on machine i. The abbreviations used can be summarized along the following lines:

$$
\begin{aligned}
j &= \text{job index;}\\
i &= \text{machine index;}\\
q &= \text{operation index;}\\
O(j) &= \text{total number of operations of job } j\text{;}\\
Z_j &= \text{priority index of job } j \text{ at time } \tau\text{;}\\
\tau &= \text{current time at which the dispatching decision is to be made;}\\
p'_{qj} &= \text{processing time of the } q^{th} \text{ operation of job } j\text{, which corresponds to a certain}\\
&\quad\; p_{ij}\text{-value according to the machine } i \text{ on which operation } q \text{ is to be processed;}\\
\overline{P} &= \text{average processing time of all jobs waiting in line;}\\
P_{q+1,j} &= \text{total processing time of all jobs in the queue of the next operation } (q+1) \text{ of job } j\text{;}\\
C_{qj} &= \text{completion time of the } q^{th} \text{ operation of job } j\text{;}\\
d_j &= \text{due date of job } j\text{;}\\
c &= \text{due date allowance factor;}\\
r_j &= \text{release (arrival) time of job } j \text{ on the shop floor;}\\
r_{ij} &= \text{release (arrival) time of job } j \text{ at the current machine } i\text{;}\\
s_{ij} &= \text{setup time of operation } O_{ij} \text{ of job } j \text{ on machine } i\text{;}\\
\eta_i &= \text{utilization level of machine } i\text{;}\\
k &= \text{exponential look-ahead parameter;}\\
W_{qj} &= \text{expected waiting time of the } q^{th} \text{ operation of job } j\text{;}\\
W_j &= \text{total expected waiting time of job } j \text{ for all unscheduled operations } (> q) = \sum_{t=q+1}^{O(j)} W_{tj}\text{;}\\
W_{q+1,j} &= \text{expected waiting time of job } j \text{ at the machine of its next operation } (q+1)\text{;}\\
s_j &= \text{slack value of job } j = d_j - \tau - \sum_{t=q}^{O(j)} p_{tj}
\end{aligned}
$$

2 Single priority rules

1. 2PT = ***Double Processing Time***

$$\min \quad Z_j = 2p'_{qj}$$

2. ATC = ***Apparent Tardiness Cost***

$$\max \quad Z_j = (\frac{1}{p'_{qj}}) \left\{ \exp \left\{ -\max \left(\frac{d_j - \sum_{t=q+1}^{O(j)} (W_{tj} + p'_{tj}) - \tau - p'_{qj}}{(k\,\overline{P})}, 0 \right) \right\} \right\}$$

3. AVPRO = ***Average Processing time per Operation***

$$\min \quad Z_j = \sum_{t=1}^{O(j)} \frac{p'_{tj}}{O(j)}$$

4. COVERT = ***Cost Over Time***

$$\max \quad Z_j = \begin{cases} \frac{1}{p'_{qj}} \frac{(W_j - s_j)}{W_j} & \text{if} \quad 0 \leq s_j < W_j \\ 0 \quad \text{if} & s_j \geq W_j \\ \frac{1}{p'_{qj}} & \text{if} \quad s_j < 0 \end{cases}$$

5. CR = ***Critical Ratio***

$$\min \quad Z_j = \frac{d_j - \tau}{\sum_{t=q}^{O(j)} p'_{tj}}$$

6. EDD = ***Earliest Due Date***

$$\min \quad Z_j = d_j$$

7. FDD = ***Flow Due Date***

$$\min \quad Z_j = r_j + \sum_{t=1}^{q} p'_{tj}$$

8. FIFO = ***First in First out***

$$\min \quad Z_j = r_{ij}$$

9. LIFO = *Last in First out*

$$\max \quad Z_j = r_{ij}$$

10. LPT = *Longest Processing Time*

$$\max \quad Z_j = p'_{qj}$$

11. LWKR = *Least Work Remaining*

$$\min \quad Z_j = \sum_{t=q}^{O(j)} p'_{tj}$$

12. MCOVERT = *Modified Cost Over Time*

$$\max \quad Z_j = \begin{cases} \frac{1}{p'_{qj}} \frac{(W_j - s_j)}{W_j} \quad \text{if} \quad 0 \leq s_j < W_j \\ 0 \quad \text{if} \quad s_j \geq W_j \\ \frac{-s_j}{p'_{qj}} \quad \text{if} \quad s_j < 0 \end{cases}$$

13. MOD = *Modified Operational Due date*

$$\min \quad Z_j = \max(r_j + c \sum_{t=1}^{q} p'_{tj}; \tau + p'_{qj})$$

14. MOPNR = *Most Operations Remaining*

$$\max \quad Z_j = O(j) - q + 1$$

15. MWKR = *Most Work Remaining*

$$\max \quad Z_j = \sum_{t=q}^{O(j)} p'_{tj}$$

16. NPT = *Next Processing Time*

$$\min \quad Z_j = p'_{q+1,j}$$

17. OPFSLK/PT = *Operational Flow Slack per Processing Time*

$$\max \quad Z_j = \frac{\max(\tau + p'_{qj} - (r_j + \sum_{t=1}^{q} p'_{tj}); 0)}{p'_{qj}}$$

18. PW = *Process Waiting time*

$$\min \quad Z_j = \tau - C_{q-1,j}$$

19. RR = *Raghu and Rajendran*

$$\min \quad Z_j = \frac{(s_j \ \exp(-\eta_i) \ p'_{qj})}{\sum_{t=q}^{O(j)} p'_{tj}} + \exp(\eta_i)p'_{qj} + W_{q+1,j}$$

20. SL = *negative slack*

$$\min \quad Z_j = \min(s_j; 0)$$

21. Slack = *slack*

$$\min \quad Z_j = s_j$$

22. Slack/OPN = *Slack per Remaining Operations*

$$\min \quad Z_j = \begin{cases} \frac{s_j}{O(j)-q+1} & \text{if} \quad s_j \geq 0 \\ s_j \times (O(j) - q + 1) & \text{if} \quad s_j < 0 \end{cases}$$

23. Slack/RPT = *Slack per Remaining Processing Time*

$$\min \quad Z_j = \frac{s_j}{\sum_{t=q}^{O(j)} p'_{tj}}$$

24. SPT = *Shortest Processing Time*

$$\min \quad Z_j = p'_{qj}$$

25. SPT/MWKR = *Shortest Processing Time per Work Remaining*

$$\min \quad Z_j = \frac{p'_{qj}}{\sum_{t=q}^{O(j)} p'_{tj}}$$

26. SS = *Shortest Setup time*

$$\min \quad Z_j = s_{ij}$$

27. WINQ = *Work In Next Queue*

$$\min \quad Z_j = P_{q+1,j}$$

28. WSL = *Waiting Slack*

$$\min \quad Z_j = \min(s'_j; 0)$$

$$\text{with} \quad s'_j = d_j - (\tau + P_{q+1,j} + \sum_{t=q}^{O(j)} p'_{tj})$$

3 Combined priority rules

The combined priority rules (with "+"-sign) are obtained by summing up the different priority values. For example, the priority value for the SPT +WINQ+ SL-rule (Holthaus and Rajendran, 1997) is an additive combination of the processing time of the job, the work content of all jobs in the queue of the next operation of the job and its non-positive slack value and is obtained according to following formula:

$$\min \quad Z_j = p'_{qj} + P_{q+1,j} + \min(s_j; 0)$$

By combining priority rules, several job characteristics are considered simultaneously when developing priorities. As such, the overall shop performance can be improved. Some other combined rules discussed in the literature and used in table **??** are SPT+WINQ (Holthaus and Rajendran, 1997), 2PT+WINQ+NPT, SPT+WINQ+NPT+WSL (Holthaus and Rajendran, 2000), CR+SPT, Slack/RPT+ SPT (Anderson and Nyirenda, 1990) LWKR+SPT (Dominic et al., 2004), SPT+PW and SPT+PW+FDD (Jayamohan and Rajendran, 2000). Our newly develop priority rules follow the same combination principle.

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