Appendix C Tolerance limits for the multivariate schedule control metrics

Hotelling's T^2 statistic is conjectured to scale with an F distribution according to [1]. Tolerance limits for this schedule control metric can be found using equation (C1), where $F_\alpha(k, n-k)$ is the critical value for an F distribution with k and $n-k$ degrees of freedom and a tolerance level α.

$$T_\alpha^2(k) = \frac{(n-1)(n+1)k}{n(n-k)} F_\alpha(k, n-k) \tag{C1}$$

[1] has also shown that the SPE metric follows a weighted Chi-squared distribution $g\chi_h^2$, within a reasonable approximation, if the assumption for the multivariate normality of $\mathbf{x}$ holds. The weight (g) and the degrees of freedom (h) of $g\chi_h^2$ can then be both expressed as functions of the loadings matrix P. For convenience, and with a proven effectiveness, g and h can also be estimated based on the method of matching moments between a $g\chi_h^2$ distribution and the distribution of SPE for the reference matrix. The mean and variance of the $g\chi_h^2$ ($\mu = gh$,$\sigma^2 = 2g^2h$) are then estimated by the sample mean (m) and variance (v) of the SPE sample, provided that the number of SPE observations is sufficiently large. A tolerance limit for SPE with a tolerance level α can then be found using equation (C2), where $\chi^2_{2m^2/v,\alpha}$ is the critical value of the chi-squared distributed variable with $2m^2/v$ degrees of freedom at a tolerance level α.

$$SPE_\alpha = (v/2m)\chi^2_{2m^2/v,\alpha} \tag{C2}$$

References

[1] G. E. Box, Some theorems on quadratic forms applied in the study of analysis of variance problems, i. effect of inequality of variance in the one-way classification, The Annals of Mathematical Statistics 25 (2) (1954) 290–302.