

The book cover features a dark blue background with a gradient to a lighter blue at the bottom. On the left side, there are several interlocking gears of various sizes, some in dark blue and some in a lighter blue. On the right side, there is a large white starburst or asterisk shape. The text is located in the upper left and bottom left corners.

# Taking Sound Business Decisions

From Rich Data to Better Solutions  
(Second Edition)

Mario Vanhoucke

# Taking Sound Business Decisions

## From Rich Data to Better Solutions

Mario Vanhoucke

© 2017, OR-AS, Merelbeke, Belgium

[www.or-as.be/books](http://www.or-as.be/books)

September 2017 (Edition 2)

### **Mario Vanhoucke**

**Academic contact:** Professor at Ghent University, Vlerick Business School and UCL School of Management of University College London

E-mail: [mario.vanhoucke@ugent.be](mailto:mario.vanhoucke@ugent.be), [mario.vanhoucke@vlerick.com](mailto:mario.vanhoucke@vlerick.com) or [m.vanhoucke@ucl.ac.uk](mailto:m.vanhoucke@ucl.ac.uk)

Twitter: @MarioVanhoucke

**Business contact:** Partner at OR-AS

E-mail: [mario.vanhoucke@or-as.be](mailto:mario.vanhoucke@or-as.be)

Twitter: @ORASTalks

### **OR-AS bvba**

Hoorndriesstraat 10

9820 Merelbeke

Belgium

E-mail: [info@or-as.be](mailto:info@or-as.be)

Website: [www.or-as.be](http://www.or-as.be)

No part of this book may be reproduced in any form, by print, photo print, microfilm or any other means without written permission from the author.

# Taking Sound Business Decisions

## From Rich Data to Better Solutions



**“The whole art of teaching is only the art of awakening the natural curiosity of young minds for the purpose of satisfying it afterwards.” - Anatole France**



Download a print friendly PDF version of this book from [www.or-as.be/books](http://www.or-as.be/books)

# Table Of Contents

## Introduction

### Methodology

Decision making for business: What is mathematical programming?

Deterministic models

What is linear programming?

What is sensitivity analysis?

What is integer programming?

What is nonlinear programming?

Stochastic models

What is decision tree analysis?

What is Monte Carlo simulation?

What is discrete event simulation?

### Applications

Warehousing

Nurse rostering

Steel production

Job shop scheduling

Airline crew scheduling

Tunnel construction

Machine scheduling

Course assignment

Custom packs design (*new*)

### Research

Operations Research and Scheduling

Optimisation in project scheduling: Exact and heuristic methods

Optimisation in project scheduling: Multi-mode trade-offs for project activities

Optimisation in healthcare: Scheduling nurses in hospitals

Optimisation in production scheduling: Scheduling jobs on machines

Optimisation & logic: Scheduling problems with SAT solvers

Statistical project control: Advanced statistics for Earned Value Management

Controlling projects: Time/cost trade-offs and artificial intelligence

Projects contracting: Incentives for time, cost and scope (*new*)

Optimisation in project scheduling: Payment models and net present value (*new*)

Controlling projects: Exponential smoothing and reference class forecasting (*new*)

What's your story?

### Education

Introduction

Master in Operations Management

Do cool things that matter

It isn't big data but big algorithms that is going to change the world

The movie: Out now!

Course outlines

Integrated Management Exercise

Decision Making for Business I

Decision Making for Business II

Project Management

### Nice to know

Solving problems using software

Nonlinear transformations: How to model absolute values?

Nonlinear transformations: How to use the big M method?

Non-negativity constraints: Shadow prices = reduced costs? (*new*)

The more you practice, the more you can: Linear and integer programming examples

The millennium prize problems: An 'easy' way to get rich!

### **Library**

Nassim Nicholas Taleb: Embrace randomness and don't be fragile ([new](#))

Daniel Kahneman: The riddle of experience versus memory ([new](#))

Alan Turing: Meet the father of the digital world ([new](#))

Richard Dawkins: The existence of the statistically improbable ([new](#))

John Nash: Life is a game ([new](#))

Eliyahu Goldratt: Theory of constraints ([new](#))

Mario Vanhoucke: Being a project manager is the most beautiful job in the world ([new](#))

### **References**

### **Epilogue**

Dear reader,

Welcome to the book “**Taking Sound Business Decisions: From Rich Data to Better Solutions**” written for anyone with an interest in modelling and decision making.

Before you proceed, every potential reader should first run this program, written in C++. It's a simple test to get to know whether this book is valuable to you!

Mario Vanhoucke

```
bool main(void)
{
    If { printf("You live in an age of fast computers, cheap or free available software, and lots of data"); }
    then
    {
        printf("Continue reading this book");
        continue;
    }
    Else
    {
        printf("Delete this PDF from your local drive (or from the cloud)");
        goto label_remove_this;
    }

    If { printf("You completely trust your gut feeling in making decisions"); }
    then
    {
        printf("I have been successful all my life. I'm simply a prodigy");
        printf("Delete this PDF from your local drive (or from the cloud)");
        goto label_remove_this;
    }
    Else
    {
        printf("Continue reading this book");
        continue;
    }

    If { printf("You are enthusiastic about new things and you are not afraid of computer science terminology[a]"); }
    then
    {
        goto label_start_reading;
    }
    Else
    {
        goto label_remove_this;
    }

label_start_reading:
    goto next-page;
    return true;
label_remove_this:
    goto delete_button_on_your_computer;
    return false;
}
```

<sup>[a]</sup> This if-statement is superfluous if you enrolled in a university or MBA programme, since in this case you expressed that you are enthusiastic about learning new things!

# Methodology



# Decision making for business

## What is mathematical programming?

*“Whenever you see a successful business, someone once made a courageous decision” (Peter Drucker)*

### Mathematical programming

In the Mathematical Programming Glossary published by the INFORMS Computing Society, the section on “The Nature of Mathematical Programming” describes **mathematical optimisation** or **mathematical programming** as the selection of a best element (with regard to some criteria) from some set of available alternatives. It concerns the problem of optimising a function of many variables, often subject to a set of constraints.

Mathematical models in which the outcomes are precisely determined through known relationships between decision variables and parameters, without any room for random variation in the data are known as **deterministic** models. Alternatively, **stochastic** models make use of ranges of values for the parameters and assume that the input data is uncertain, mostly modelled in the form of probability distributions.

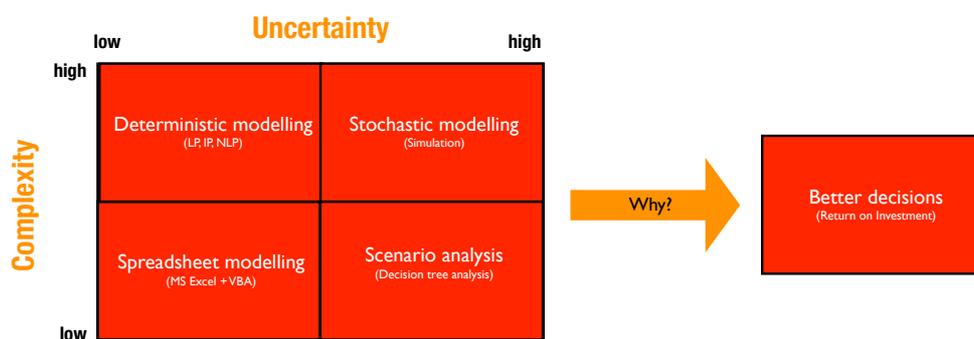
Deterministic models include:

- Linear programming (LP)
- Integer programming (IP)
- Nonlinear programming (NLP)

Stochastic models include:

- Decision tree analysis (DTA)
- Monte Carlo simulation (MCS)
- Discrete event simulation (DES)

In this “**What is mathematical programming?**” section of this book, the various techniques and methodologies that all support better decision making will be briefly explained. While the deterministic models can be used to solve highly complex problems, they are not very good in optimising highly uncertain systems. The stochastic models, on the contrary, can cope with both simple and complex environments with a high degree of uncertainty.



## Decision making for business

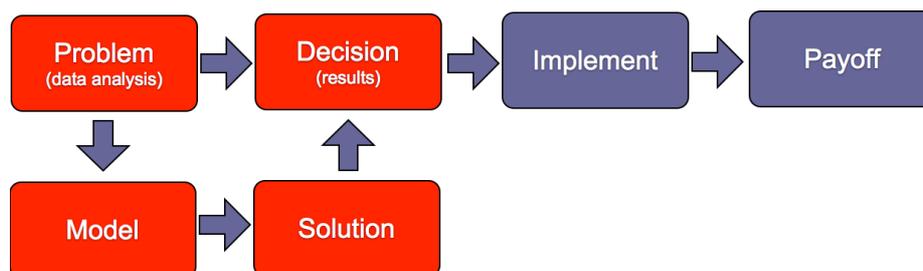
While mathematical programming originates from the computer science discipline and has long been the playground for mathematicians and computer specialists, its main purpose is to enable decision makers to facilitate and improve their decisions based on a clever combination of company data and their often rich experience using the toolkit available from the Operations Research community. One of the most well-known organisations that promote greater use of this Operations Research knowledge to the general public is INFORMS. The Institute for Operations Research and the Management Sciences ([www.informs.org](http://www.informs.org)) is an international society for practitioners in the fields of Operations Research (OR) and Management Science (MS). Through their publications in scholarly journals, they clearly show that Operations Research is much more than a quantitative playground for mathematicians. Instead, the methodology is becoming a standard in the business analytical world, and it will become even more important as technologies advance.

The methodology is useful for ...

- ... **business analysis**: Operations Research is no longer a “black art” practiced only by mathematicians, consultants and computer specialists, but instead a methodology used to solve real business problems. Data and algorithms have a tendency to outperform human intuition in a wide variety of circumstances, leading to an industry that is more productive and more profitable.
- ... **trade-off analysis**: The methodology becomes increasingly useful when one is faced with a situation involving conflicting alternatives. Given limited resources and the fast speed of business decisions, analysing trade-offs is inherent to any business decision, and the central question often is “How much must I give up to get a little more of what I want most?”
- ... **big data analysis**: Data-driven companies treat their data as a corporate asset and leverage it for competitive advantage. Successful business analytics depend not only on data quality, but also on skilled analysts who understand the technologies and methodologies and who show a clear commitment to data-driven decision making.

## Approach

An analytical approach to decision making requires a thorough understanding of the business problem and a translation of this problem into a mathematical model. The analysis of data is a process of inspecting, cleaning, transforming, and modelling data with the goal of discovering useful information to be transformed into a model that reflects a simplified version of reality. Its solution should consist of an ideal scenario and should enable suggesting conclusions, and supporting decision making that could be implemented and lead to an improved payoff for the business.



## What is a good model?

A model is good, relevant and useful when it reflects reality to a certain degree and when it contributes to a higher payoff that couldn't be obtained without this model. The real purpose of a model is to act only as a **predictive model**, and it should be a tool to **support decisions** rather than to make decisions. It should help to guide the human decision maker into the right decision, hereby potentially exploring directions that never could be found without the help of the model. Therefore, the real value of a model is measured by the **return on investment (ROI)** it generates when implementing the decisions made based on its solutions. Without this improved payoff, a model is simply useless, and one should go back to his/her intuition or any other means to make decisions.



# Deterministic models

## What is linear programming?

*“Programming is modellisation of real life” (Fode Toure)*

### Linear programming

**Linear programming (LP)** is a mathematical modelling method to achieve the best outcome (such as maximum profit, safety and customer satisfaction or, alternatively, minimum cost, waste and idle time) and contains requirements and limitations that are represented by linear relationships. The outcome that is to be optimised is known as the objective, while the various requirements and limitations are represented by the constraints.

#### Inputs

- **Decision variables.** A set of quantities that need to be determined in order to solve the problem. They express what exactly needs to be found to solve the problem. The values of the variables are unknown, and the problem is solved when the best values of the variables have been identified.
- **Parameters.** The known data available in the company to support the decision making process, expressed as profit margins, capacity limits, productivity values and much more. The parameters are used to model the objective function and various constraints and are assumed to represent correct values.

#### Model

- **Objective function.** Defines how the model will be evaluated and how the return on investment will be measured. It is the goal the decision maker wishes to optimise while generating a solution for the problem. It is defined as a minimum target value (e.g. cost) or maximum target value (e.g. profit).
- **Constraints.** The limitations or requirements inherent to the problem, defined as a minimum requirement (e.g. quality) or maximum (e.g. capacity) requirement or sometimes as a strict equality requirement (e.g. exactly 10 tons need to be produced).

#### Outputs

- **Solution.** The best values of the decision variables, returned by the software used to solve the model, are now no longer unknown and constitute the model solution.
- **Objective value.** The numerical value for the objective function expressed as a single number.
- **Sensitivity report.** Since a solution for a model is depending on the input data (parameters), it often does not immediately provide a solution to the business problem. Due to unexpected changes in these parameters, the real solution can often be somewhat different. This margin of error is measured by a sensitivity report that is automatically generated for each solution (cf. “What is sensitivity analysis?” in this book).

### How

Newcomers in modelling and data-driven decision making often don't know where and how to start. They often struggle with the new terminology and keep stuck in the mathematical equations and software tools. Therefore, below I propose a simple four-step approach with a focus on the problem formulation rather than on the modelling of the equations, and although being oversimplified, it forces you to carefully think about a problem in all its facets, without bothering too much about technical skills and mathematical details. Moreover, the first three steps can be followed without any modelling background since they primarily focus on understanding the business problem and finding out what exactly needs to be done to provide a good suggestion to the business people in charge of the problem.

#### Step 1. What do I have to determine?

This is often the most important question, since it defines what the problem is that needs to be solved. Providing a detailed answer on this question (in words!) often requires a deep understanding of what exactly needs to be done to help the company. It defines your decision variables for your model of step 4.

## Step 2. What are the relations?

In this step, you learn what the requirements and limitations are what makes your problem so difficult. Without relations, there shouldn't be a need for building the model as the solution is then too straightforward and easy to solve by just looking at the problem. The answer on this question (again, do it in words!) defines the constraints in your model.

## Step 3. What is the goal?

Since modelling is the art of doing better, you should define a goal to reach, which is an objective to be optimised. A model requires a single objective to be minimised (such as cost) or maximised (such as profit).

## Step 4. Translate words into symbols

This is the easiest step for a modeller with experience, but often the hardest nut to crack when you are a rookie in modelling. In this step, all answers to the three previous questions should now be translated into mathematical symbols, and once done, should be loaded into a software tool to find a solution. Despite the wide variety of software tools available, solving a model with MS Solver is the ideal start! It's easy, fast and free (student version) and it runs on Windows and Mac (cf. "Solving problems using software" in this book).

# EXAMPLE

Consider a simple example from an online document that I have found on the website of University of Nebraska - Lincoln about an oil refinery producing two products: jet fuel and gasoline. The profit for the refinery is €0.10 per barrel for jet fuel and €0.20 per barrel for gasoline. The following conditions must be met:

1. Only 10,000 barrels of crude oil are available for processing.
2. The refinery has a government contract to produce at least 1,000 barrels of jet fuel.
3. The refinery has a private contract to produce at least 2,000 barrels of gasoline.
4. Both products are shipped in trucks, the delivery capacity of the truck fleet is 180,000 barrel-miles.
5. The jet fuel is delivered to an airfield 10 miles from the refinery.
6. The gasoline is transported 30 miles to the distributor.

How much of each product should be produced for maximum profit?

The model is straightforward and contains only two decision variables. Note that real models might need thousands of these variables. But let's keep it simple for now. Let  $x_1$  represent the number of barrels of jet fuel and  $x_2$  represent the number of barrels of gasoline. The model is then formulated as follows:

$$\begin{aligned} &\text{maximise profit } (P) = 0.10 x_1 + 0.20 x_2 \\ &\text{subject to the following constraints:} \\ &x_1 + x_2 \leq 10,000 \\ &x_1 \geq 1,000 \\ &x_2 \geq 2,000 \\ &10 x_1 + 30 x_2 \leq 180,000 \\ &x_1 \geq 0 \text{ and } x_2 \geq 0 \end{aligned}$$

The model can be easily solved using a commercial solver, such as MS Solver that is a plug-in for MS Excel. Doing so, the optimal solution will be returned as  $x_1 = 6,000$  barrels and  $x_2 = 4,000$  barrels, resulting in a total profit  $P = €1,400$ . The optimal value for the decision variable are all integer values and no fractional barrels will be produced. But that is only a coincidence, since it could have been fractional too. Just change the 0.20 into 0.30 in the objective function and solve the problem again. You will produce 5.666.67 barrels of gasoline. A fractional barrel? Yes, that's what you might get when using linear programming.

Note that the total number of barrels is exactly 10,000 since it is not allowed to produce more than this due to the first constraint. We call this first constraint a binding constraint since its left hand side ( $x_1 + x_2$ ) is exactly equal to its right hand side (10,000). The fourth constraint is also a binding constraint, for the very same reason. Binding constraints prevent to have a solution better than the one currently found. The three other constraints are not binding and have currently not much impact on the solution. For these constraints, the right hand side is different from the left hand side, and this

difference is called slack. Take e.g. constraint 2 with a right hand side value of 1,000 and a left hand side value of  $x_1 = 6,000$  and hence a slack value of 5,000. Knowing which constraints are the binding constraints can be very helpful to understand your problem. I would call it “bottleneck detection”. Got it?

Objective Cell (Max)					
Cell	Name	Original Value	Final Value		
\$E\$6	Objective	0.000 €	1,400.000 €		

Variable Cells					
Cell	Name	Original Value	Final Value	Integer	
\$C\$8	Decision Variables x1	0	6000	Contin	
\$D\$8	Decision Variables x2	0	4000	Contin	

Constraints					
Cell	Name	Cell Value	Formula	Status	Slack
\$E\$10	Constraints	10000.00	\$E\$10<=\$G\$10	Binding	0
\$E\$11		6000.00	\$E\$11>=\$G\$11	Not Binding	5000.00
\$E\$12		4000.00	\$E\$12>=\$G\$12	Not Binding	2000.00
\$E\$13		180000.00	\$E\$13<=\$G\$13	Binding	0

Answer report for problem 1

The binding/non-binding status of the constraints, the slack values and of course the optimal solution is available in MS Solver by looking at the Answer Report that is available in the picture above. But much better is to plug in this model into MS Excel yourself, and solve it using MS Solver (cf. “Solving problems using software” in this book). The model is available for download at the link mentioned below.



Download the problem (Tab “Problem 1”) from [www.or-as.be/books/TSBD\\_Examples.xlsx](http://www.or-as.be/books/TSBD_Examples.xlsx)

**Nice to know...**

Using linear programming requires some attention since it is not such a flexible technique. Similar to statistical techniques or artificial intelligence algorithms, it relies on some strict assumptions to follow. It requires some specialised knowledge that you have to understand (and once you do, you will never forget) to avoid the misuse of the LP technique. A summary is given below:

- Inequality constraints ( $\leq$  or  $\geq$ ) and equality constraints (=). There is a general tendency to try to avoid the equality constraints, in favour of inequality constraints, but you can use them whenever necessary.
- Non-negativity constraints: For technical reasons, the decision variables of linear programs must always take non-negative values (i.e., they must be greater than or equal to zero). In most cases, this non-negativity requirement will be reasonable and even necessary. In other cases, when variables might take negative values, there are tricks available to overcome this problem, but this is beyond the scope of this LP introduction. As for now, we consider the “extra constraints” are necessary and you should always include them. Most often, software tools incorporate them automatically after a single click on the “Add non-negativity constraints” button. In the previous model, I explicitly mentioned them in the last set of constraints.
- Solutions obtained by the model can be roughly classified into three classes, as follows:
  - Infeasible solutions: As these solutions do not satisfy all the constraints, they make little sense and are not worth considering (at least one constraint is violated).
  - Feasible solutions: These solutions satisfy all constraints and might therefore be valuable solutions to the company. However, since modelling is “the art of doing better” and aims at improving the current existing (and thus feasible) solutions of the company, finding just a feasible solution is not the desired outcome of a model.

- Optimal solutions: This solution is the best possible feasible solution that can be obtained with the constraints and parameters of the model. This is the solution that you want to show to the company, since it is quite likely better than the existing (feasible) solution that they now currently have. Don't bother too much whether this solution will be the one that they really like to implement, as for now, it is a good starting point for a discussion since it is the best-case solution for the problem as you defined it in collaboration with the company. We will later put that solution into question, in the "What is sensitivity analysis?" part of this book.
- Linear functions: All constraints as well as the objective function have to be linear equations. A linear equation consists of a sequence of numbers multiplied by a decision variable (e.g.  $5x_1 + 6x_2$  with  $x_1$  and  $x_2$  the two decision variables). All deviations from this strict assumptions can be considered as nonlinear and therefore not appropriate for the LP techniques. Examples of nonlinear equations with the two decision variables  $x_1$  and  $x_2$  are given below:
  - $5x_1^2 + 6x_2^2$ : Don't square decision variables, it's not linear!
  - $5x_1 / x_2$ : Don't divide decision variables, it's not linear!
  - $\max\{x_1, x_2\}$ : This equation contains a word (max), which isn't linear either!
 Later in this book, various tips for nonlinear to linear transformations are given.

# Deterministic models

## What is sensitivity analysis?

*“Stay committed to your decisions, but stay flexible in your approach” (Tony Robbins)*

### Sensitivity analysis

Since any model is a careful abstraction of a real system, it seldom describes every little detail of the often complex and changing reality. The parameters used to assemble the model's constraints and objective function often come from databases and other sources, and are therefore very prone to errors or changing circumstances that are or cannot be captured by the model. Therefore, in order to get an idea of how much each input (parameter) is contributing to the output (objective) uncertainty, any modeller should have access to a technique that provides an evaluation of the confidence of the model. Sensitivity analysis is a toolkit and an essential ingredient when using linear programming and suits very well for that purpose.

Wikipedia describes the technique as the study of how the uncertainty in the output of a mathematical model or system (numerical or otherwise) can be apportioned to different sources of uncertainty in its inputs ([en.wikipedia.org/wiki/Sensitivity\\_analysis](https://en.wikipedia.org/wiki/Sensitivity_analysis)). It provides a way to show how a model's results would be affected, and how responsive or sensitive those results would be, to changes in the values of specific parameters.

While there exist a wide range of types of sensitivity analysis that can be used for any modelling technique, the LP technique is equipped with a set of metrics, known as the shadow prices and reduced costs, that belong to the class of partial sensitivity analysis. They allow the modeller to select a single parameter, change its value while holding the values of other parameters constant, and see how much the model's results change in response.

### Shadow price

[Associated with each constraint is a shadow price value].

In linear programming, the shadow price is the instantaneous change in the objective value of the optimal solution obtained by changing the right hand side constraint by one unit. In other words, it is the marginal utility of relaxing the constraint, or, equivalently, the marginal cost of strengthening the constraint.

Assume a linear constraint  $5x_1 + 6x_2 \leq 100$ , where the left hand side [LHS] is a linear equation of two decision variables and the right hand side [RHS] a known parameter value. Changing the value of the RHS of that specific constraint from 100 to say 101 (one unit) will probably have an impact on the optimal value of the objective value. That impact is measured by the shadow price for that constraint.

Still confused? Here is what it means on an example. It's pretty intuitive.

For a maximisation problem, the constraints can often be thought of as restrictions on the amount of resources available, and the objective can be thought of as profit. Then the shadow price associated with a particular constraint tells how much the optimal value of the objective would increase per unit increase in the amount of resources available. Consequently, the shadow price associated with a resource denotes the potential profit increase one would get by increasing the amount of that resource by one unit. Or in other words, it shows how much the modeller would be willing to pay for an additional resource. Highly relevant information!

### Reduced cost

[Associated with each variable is a reduced cost value].

In linear programming, the reduced cost is the amount by which an objective function parameter would have to improve before it would be possible for a corresponding variable to assume a positive value in the optimal solution. A reduced cost is associated with each decision variable in the model, and has only a meaningful value for variables for which the optimal value is positive (not zero).

Since the reduced cost value indicates how much the objective function coefficient on the corresponding variable must be improved before the value of the variable will be positive in the optimal solution, its meaning differs along the type of objective in the model [minimisation or maximisation]. Improving the objective for maximisation problems means increasing its value, while it refers to decreasing values for minimisation problems.

Consider a simple intuitive example.

Consider a cost minimising model with an objective “minimise  $100 x_1 + 150 x_2$ ” and an optimal solution  $x_1 = 50$  and  $x_2 = 0$ . The decision variables can e.g. express the amount of products of type 1 and type 2 that will be produced on a weekly basis, and the objective coefficient parameters 100 and 150 express the unit cost of production. Clearly, the production amount of type 2 is zero, indicating that the cost of production is too high relatively to the first product type. But what if you wish to produce this second type product anyway? You should reduce your cost by ... indeed ... the value of the reduced cost, which is at least €50.

Alternatively, for a profit maximisation problem, the reduced cost, or opportunity cost, would represent the amount of money by which the price of the product will have to increase in order to make it profitable.

The reduced cost gives for each variable which is currently zero [ $x_2$ ] an estimate of how much the objective function will change if we make [force] that variable to be non-zero. If the optimal value of a variable is positive [not zero], as is the case for  $x_1$ , then the reduced cost is always zero.



Consider the example oil refinery producing example discussed earlier. After solving the problem, the answer report was automatically generated showing the optimal solution [ $x_1 = 6,000$  and  $x_2 = 4,000$  with a profit of €1,400]. Next to the answer report, the user also has the option to generate a “sensitivity report” that gives the shadow prices and reduces costs.

Variable Cells						
Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$C\$8	Decision Variables x1	6000	0	0.1	0.1	0.033333333
\$D\$8	Decision Variables x2	4000	0	0.2	0.1	0.1

Constraints						
Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$E\$10	Constraints	10000	0.05	10000	4000	3333.333333
\$E\$11		6000	0	1000	5000	1E+30
\$E\$12		4000	0	2000	2000	1E+30
\$E\$13		180000	0.005	180000	100000	40000

Sensitivity report for problem 1

The shadow prices for the 4 constraints are as follows:

- Shadow price constraint 1: 0.05
- Shadow price constraint 2: 0
- Shadow price constraint 3: 0
- Shadow price constraint 4: 0.005

Consider e.g. the first constraint, with a right hand side of 10,000 and a shadow price of €0.05. Each time the right hand side of constraint is changed by one unit, upwards or downwards from 10,000 (current value) to 10,001 or 9,999, the profit P will increase or decrease from €1,400.00 (current best solution) to €1,400.05 or €1,399.95. Hence, if someone wishes to increase the profit P by €100, the right hand side of this constraint should be increased to 2,000, or in other words, the total number of barrels should no longer be restricted to 10,000 but to 12,000 barrels. Note that this profit increase is not endless, since other constraints might become binding. That's why the concept of "allowable increase" (4,000) and "allowable decrease" (3,333.33) is added to each sensitivity report. It shows the range in which the "€0.05 per unit change" is valid.

The reduced cost is a similar interesting concept, but cannot be illustrated on the example above since both reduced costs for variables  $x_1$  and  $x_2$  are equal to zero! The reason is that the values for the  $x_1$  and  $x_2$  variables are positive in the optimal solution ( $x_1 = 6,000$  and  $x_2 = 4,000$ ) and reduced costs are only meaningful when the optimal values are equal to zero.

But consider another, very straightforward, yet illustrative example using another small LP model as shown below:

$$\begin{aligned} &\text{minimise cost } [C] = 10 x_1 + 7 x_2 \\ &\text{subject to the following constraints:} \\ &x_1 + x_2 \geq 10 \\ &x_1 \geq 0 \text{ and } x_2 \geq 0 \end{aligned}$$

It is easy to find the optimal solution, even without using a software tool. The optimal values are equal to  $x_1 = 0$  and  $x_2 = 10$ , resulting in the minimum cost objective value of €70. For example, this could mean that no production is scheduled for product type 1, only 10 units are produced for product type 2, resulting in the minimum cost of €70. The readers who wish to solve this problem in MS Solver can find this model under the tab "Problem 2" in the same MS Excel file as for problem 1. Since the optimal value of the  $x_1$  variable is now equal to zero, the concept of reduced cost now has a meaning. The reason why no production is proposed is that the product type 1 is too expensive compared to its alternative product type 2. However, assume you wish to produce the product type 1 anyway, for strategic reasons, or maybe just because you like it, then by what amount should you have to reduce the cost (indeed, reduced cost) such that it can compete with its alternative? Indeed, the cost should go down from €10 per unit to €7 per unit, or lower. That is a minimum decrease of €3. Well, look at the sensitivity report below: the reduced cost of  $x_1 = 3$  while the reduced cost for  $x_2$  has no real meaning, since its value is positive.

Variable Cells						
Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$C\$8	Decision Variables x1	0	3	10	1E+30	3
\$D\$8	Decision Variables x2	10	0	7	3	7
Constraints						
Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$E\$10	Constraints	10	7	10	1E+30	10

Sensitivity report for problem 2

Not impressed? Of course not. You could easily find the answer without the report. But assume you now produce a set of products that runs in the thousands. Each variable (product) that is not produced gives you a reduced cost. And that's impressive and important information!



Download the problem (Tab "Problem 2") from [www.or-as.be/books/TSBD\\_Examples.xlsx](http://www.or-as.be/books/TSBD_Examples.xlsx)

## Did you know ...

In the previous, it is shown that

- A shadow price value is associated with each constraint of the model
- A reduced cost value is associated with each variable of the model

Then it should be intuitively clear that

- The reduced cost is equal to the shadow price of the non-negativity constraint of the variable.

Really, think about this for a minute or two. If you understand this feature, a sensitivity analysis should not have any secrets anymore.

# Deterministic models

## What is integer programming?

*“Integer programming is useful, fast, and cheap. If that isn’t sexy (for an algorithm!), then I don’t know what is.” (Michael Trick)*

### Integer programming

**Integer programming (IP)** is a mathematical modelling method similar to linear programming but some or all of the decision variables are now restricted to be integers.

Several reasons exist why decision variables must have integer values. The variables might represent quantities such as the number of mobile phones produced per week or the number of pilots scheduled this month, or the variables represent a decision that must be taken {1} or not {0}. Therefore, IP modelling has numerous applications in practice, such as production scheduling, personnel assignments, supply chain management, logistics, telecommunication, project scheduling and much more. Integer programming can be used in various ways and three classes can be distinguished, as follows:

- Integer programming (IP): All variables must have integer values.
- Mixed integer programming (MIP): Some variables must have integer values, other can still have fractional values.
- Binary integer programming (or 0/1 IP): Some or all of the variables must have integer values that are restricted to only 0 or 1 values.

But you shouldn’t worry too much about the differences, since (M)IP and 0/1IP models can all be solved in the same way. You just need a (powerful) computer, some expertise and a software tool to upload your model.

### LP or IP?

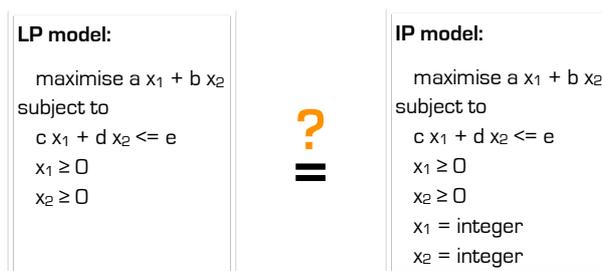
So, the difference between LP and IP lies in the integrality requirement of the decision variables, and that’s it? Well, yes... and no. There are some similarities, but there also are some differences, and it’s good to understand what they are.

Both models are methods to solve problems and support decisions, i.e. both have

- The best outcome: Both models lead to the best possible objective (i.e. the optimal solution).
- Linear constraints and objectives: Yes, that restriction holds for both models. IP also requires that all constraints (other than the integer constraints) and the objective function are linear.

But there’s a difference

- The solution must be integer, while LP allows fractional values. In a production scheduling problem, 3.2 cars cannot be produced, so the decision variables that represent the quantity of cars produced must be set to integer.
- No sensitivity report: Unlike LP, the IP models will not provide any way to assess the sensitivity of the solution. That’s a pity, since these reports are powerful. But there are techniques to cope with this shortcoming (e.g. the simulation methods discussed later).



So, as a conclusion, an LP model plus an additional set of (integer) constraints equals an IP model. It’s not a huge difference, but it has a tremendous impact on the hardness of the problems that they can solve.

## IP is not easy

IP problems generally are much more complicated than LP problems. Many of the IP classes are known as NP-hard problems (Nondeterministic Polynomial-time hard), which need very long processing times for computers to find the optimal solution. It can take months to years to solve problems of a real case size.

**What?** NP-hardness is not an easy concept, but for now, just take it as “not very easy to solve, on the contrary!”. I once heard that an NP-hardness problem is a problem that is harder than any other problem you have solved in your life. It’s not a very scientific nor accurate definition, but it enables you to grasp the idea. If you want to know more, just read the biography of **Alan Turing** on any website (e.g. [http://en.wikipedia.org/wiki/Alan\\_Turing](http://en.wikipedia.org/wiki/Alan_Turing)). It’s an interesting and fascinating story! It’s also a sad story.

**Worthless?** You don’t have months or years to wait for a solution, since the decision that you must take is urgent? Don’t throw away this book and just read further, since there is still hope that your IP models might be a valuable tool for your decision making process. Indeed, with the increasing availability of data, the much faster computers, the algorithmic improvements and the numerous implementation successes that have led to many orders of magnitude speed increases in integer programs, it wouldn’t be a good idea to just skip the technique and go back to your gut feeling when taking decisions. Over the last decades, mathematical programming solvers have become more than a million times faster and lots of instances formerly out of reach can now be solved routinely.

Of course, solving integer programming models is very, very, yes, very hard in the worst case, but on average, they can lead to solutions in a reasonable time. By the way, there are many situations in which getting a solution within e.g. 5% of the optimal solution is good enough for your problem. You can always set such a **tolerance gap** as an easy way to speed up the search to minutes or hours rather than months or years, without losing much solution quality.

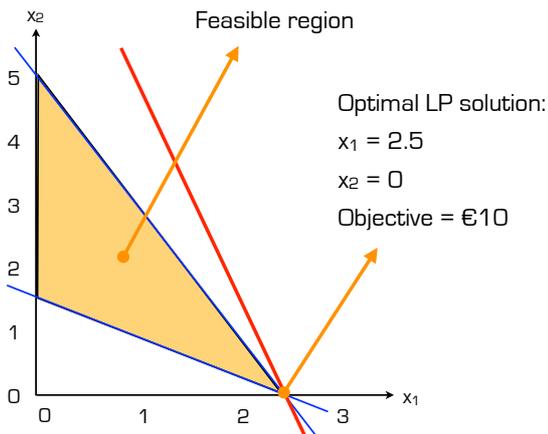
**Why?** Do you wonder why solving IP models is so much more difficult than solving LP models? There is, after all, not much difference between the two, except for the integrality constraints. Besides, the additional constraints should make the problem easier to solve, since the feasible region (i.e. the set of possible solutions that satisfy all the constraints) is much smaller, and therefore the optimal solution should be easier to find. Well ... no.

The basic idea and reason why an IP problem is more complex to solve than an LP problem can be illustrated using a simple example with only two decision variables as shown below. The problem is a maximisation problem and now contains a new set of constraints, forcing the two variables  $x_1$  and  $x_2$  to be integer values.

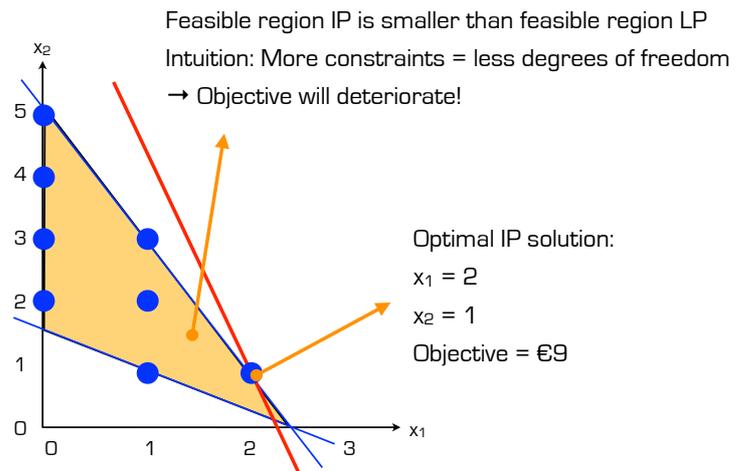
$$\begin{aligned} &\text{Maximise } 4x_1 + x_2 \\ &\text{subject to the constraints:} \\ &2x_1 + x_2 \leq 5 \\ &2x_1 + 3x_2 \geq 5 \\ &x_1 \geq 0 \text{ and } x_2 \geq 0 \\ &x_1 \text{ and } x_2 \text{ integer} \end{aligned}$$

The feasible region of this problem can be represented in a two-dimensional space (i.e. one dimension for each variable) as shown in the left figure below. Each constraint is a line (two dimensions) and the values of the decision variables are displayed on the X-axis (variable  $x_1$ ) and Y-axis (variable  $x_2$ ). Guess how this should look like for a real problem with thousands of variables. A +1000-dimensional space? Looks like something for a new Steven Spielberg science fiction movie!

## LP problem



## IP problem



**Linear programming:** All dots in the feasible region are potential candidate solutions (called feasible solutions) and probably one of these dots is already in use in the company. But you want to do better. You want the best (optimal) solution. But there are an infinite amount of dots since all fractional values are allowed, so how does an optimisation tool decide which one is the best? Well, here is where the beauty of LP comes in, since it is known that the optimal solution is always a so-called extreme point, i.e. a point where two or more constraints meet each other. Hence, this leads to a huge reduction of possible points to evaluate for being the optimal solution. That may be still run far over the thousands, but at least, it is less than infinity. Luckily, the computers are fast these days!

For the example problem above, the feasible region is shown on the left of the figure. It consists of all points between the three lines of the constraints. These constraints show the boundaries of the three constraints (constraints 1 and 2 and the non-negativity constraints, excluding the integrality constraints) assuming that the problem is an LP problem. The red line is the objective function (a line with a slope equal to  $-0.25$ ) and is maximised (i.e. the higher on the picture, the better) until the point that it just not leaves the feasible region. That particular point that lies both on the objective line and the feasible region is an extreme point where constraints 1 and 2 meet, and is known to be the optimal solution with the best possible value for the objective equal to €10. The two other extreme points,  $[x_1, x_2] = [0, 5]$  and  $[0, 1.67]$  are not optimal, since they result in a lower value for the objective function (€5 and €1.67, respectively). Obviously, the optimal LP solution  $[x_1, x_2] = [2.5, 0]$  is not the solution for the IP problem above, since its variable  $x_1$  has a fractional value.

**Integer programming:** For IP problems, the previous observation that the optimal solution always lies on an extreme point is no longer true. Only the integer dots are now part of the feasible solution. Although the number of dots has decreased dramatically, the optimal solution is no longer always an extreme point, and hence, many more points need to be evaluated. In the example, the complexity increases by almost a factor three, from 3 extreme points for LP to 8 dots for IP. Much harder. NP-harder!

The IP problem described above has an optimal solution equal to  $[x_1, x_2] = [2, 1]$  (both have integer values!) with an optimal objective function value equal to €9. The feasible region of the problem is displayed to the right of the picture, and is now reduced from a zone with an infinite amount of points to the integer dots in this zone. Only 8 solutions satisfy all the constraints, including the integrality constraints. Note that the optimal IP value (€9) is worse than the optimal LP value (€10), since the integrality constraints have reduced the size of the feasible region. More constraints result in a solution with an objective value worse than, or at the very best, equal to the problem with less constraints. No one likes many restrictions. Not even a model!



Download the problem (Tab “Problem 3”) from [www.or-as.be/books/TSBD\\_Examples.xlsx](http://www.or-as.be/books/TSBD_Examples.xlsx)

## How

There is obviously for most of us no good reason to study all technical details of algorithms that are used to solve LP and IP problems, since most of us will likely use a commercial software tool to solve the problems, having no interest in the underlying details. As long as it solves the problem! Right? However, let me mention only two well-known solution approaches that can be used to solve these problems. Details are beyond the scope of this book, but you probably already use these methods while solving problems. They are part of the well-known and easy to use MS Solver tool. And they deserve to be mentioned anyway, even without all the fancy details.

Linear programming: Linear programming models can be efficiently solved using the well-known SIMPLEX method. Often, the number of solutions in the feasible space exceeds the number of particles in the universe, however, it takes only a moment to find the optimum solution with the SIMPLEX method. The algorithm has been proposed by George Bernard Dantzig (1914 – 2005) in 1947 and is still used to solve LP models. The journal Computing in Science and Engineering listed it as one of the top 10 algorithms of the twentieth century.

Integer programming: One of the most well-known methods to solve IP problems is the branch-and-bound method. It consists basically of a clever sequence of LP models to be solved, each time splitting the problem into subproblems, and each subproblem is then treated as a new LP problem. The algorithm is searching in such a way that a tree structure is built, and deeper levels in the tree will likely result in an integer solution of the problem. However, once such a solution is found, the search needs to backtrack (i.e. return to higher levels in the tree) to search for other, hopefully better integer solutions. Hence, the optimal solution can only be found and guaranteed when all nodes in the tree are evaluated, and this can take a huge amount of time. The problem is therefore said to be NP-hard.

# Deterministic models

## What is nonlinear programming?

*“Look deep into nature, and then you will understand everything better” (Albert Einstein)*

### Nonlinear programming

A **nonlinear program** (NLP) is similar to a linear program and also consists of an objective function to be minimised or maximised and a set of constraints, but is fundamentally different since it includes at least one nonlinear function in the objective or in some of the constraints.

The problem is that nonlinear models are much more difficult to solve for numerous reasons. Unlike the tools available to solve LP and IP models, the NLP software tools all start from different starting points and may all arrive at different final solutions. Therefore, the final solution is not guaranteed to be the best one (no optimality guarantee) since they all use different algorithms and solvers for the same formulation.

While the absence of an optimality guarantee must not necessarily be a huge disadvantage in a practical setting, one must not forget that decision making is the art of doing better, and the “good but not necessarily optimal” results obtained by NLP solvers are often not good enough, even not for supporting decisions, since they do not outperform the existing company solution for the problem. Hence, in these cases, the art of doing better does not lead to improvement. There’s no single company willing to pay for a consultant who proposes a solution which is worse than the current one. No ... good enough is not always good enough!

So forget about the nonlinear equations that often occur in practice, and go back to LP and IP that are restricted to linear problems? Surely not!

### Getting solutions

Despite the complexity to solve nonlinear models, various ways exist to solve these models, including the mathematical linearisation of nonlinear equations, and the use of meta-heuristic solvers often inspired by systems from nature.

#### Solution 1. Linearise equations

Often times, nonlinear equations can be transformed to linear equations. It requires a little bit mathematical skills, but it is often not so complex and worth the effort, since linear models provide optimal solutions. As an example, the equation  $y \leq \min\{x_1, x_2\}$ , where  $y$ ,  $x_1$  and  $x_2$  represent decision variables can be transformed to a set of two constraints as  $y \leq x_1$  and  $y \leq x_2$ . While the first equation was not linear (the min operator deviates from the linearity definition), the latter two equations are 100% linear and therefore easy to use in an LP model (fractional solutions) or IP model (integer solutions).

How nice, not so difficult and elegant! And the benefits of the transformation is that the solution of an LP or IP model is optimal (best one), which is not the case for NLP. The transformation is certainly worth the effort, but unfortunately, the transformation is not always so easy. Just think about e.g. the equation  $y \leq \max\{x_1, x_2\}$  and ways to make it linear. It’s not so easy. One of the well-known methods to linearise equations like this, is known as the big M method. The M represents a very large number and the method can be used for various purposes, including the linearisation of logical constraints, absolute values, or many other nonlinearities in modelling. The use of the big M method requires some practice and experience, and examples are given later (cf. “How to use the big M method?” in this book).

## Solution 2. Imitating nature

Rather than transforming nonlinear equations to linear equations to guarantee an optimal solution for the problem, you can also simply rely on (meta-)heuristic solution procedures that solve the problem to a relatively good solution. Heuristic or meta-heuristic search procedures are designed to find a heuristic (i.e. good but not necessarily optimal) solution rather than an optimal (the best possible) solution, by performing a clever but only partial search in the solution space. They all rely on strategies that guide the search process to find near-optimal solutions, although there is no guarantee to have a maximum deviation from the (unknown) optimal solution, and therefore, the obtained quality of the solution remains unknown.

Heuristic search procedures are often classified in single solution versus population-based searches. Single solution approaches start with a (randomly) generated solution and focus on modifying and improving it using a predefined strategy. Examples are simulated annealing, iterated local search, variable neighbourhood search, and guided local search algorithms. Population-based approaches start with a set of different solutions and improve the multiple candidate solutions, often using population characteristics to guide the search that is inspired by systems observed in nature. Famous population based meta heuristics include evolutionary computation algorithms, genetic algorithms, ant colony optimisation or electromagnetic search optimisation. Most of them are **nature based approaches** and mimic processes from nature to find an equilibrium and a near-optimal solution. An yes indeed, nature is full of nonlinearities, and is apparently very good in finding solutions for problems. So the idea is not so bad. Three examples are briefly described below, and many more are available on popular websites on optimisation.

- **Genetic algorithms** are designed to simulate processes in natural systems necessary for evolution, especially those follow the principles of Charles Darwin and his "survival of the fittest.". Since in nature, competition among individuals for scarce resources results in the fittest individuals dominating over the weaker ones, this principle is used as the basis for searching better (rather than weaker) solutions in a computerised search process.
- **Ant colony algorithms** are designed for finding shortest paths that are based on the behaviour of ants searching for food. While ants initially perform a random walk in search for food, they quickly leave their markers, known as pheromones, to continuously improve their search for the fellow ants, By imitating the pheromone-based walks of ants, these algorithms increase the probability for follow paths that easily lead to the best solution (i.e. the food for ants). These types of algorithms are ideally for optimising network systems in supply chains, IT, telecommunications and much more.
- **Electromagnetism-like mechanism algorithms** follow a search pattern based on the theory of physics defined by the law of Coulomb to simulate attraction and repulsion of solutions in order to move towards more promising solutions. While the technique is less known compared to the previous search procedures, it has been successfully used in the optimisation of project and machine scheduling problems.

### How good is near-optimality?

Obviously, when meta-heuristic procedures are not able to guarantee an optimal solution, a solution is as good as the judgement of its quality for the user. But as a modeller, it is often hard to decide what the quality of the near optimal solutions is, and whether more effort is necessary to find a better solution, even closer to optimality, if it exists. But the quality of the solution provided by these types of algorithms remains unknown.

So how do you know?

You don't! But you can approximate the quality by using upper and lower bounds that can be used to validate the solution quality and support your decision to either hand over the solution to the company, or instead to throw it away and search for better alternatives.

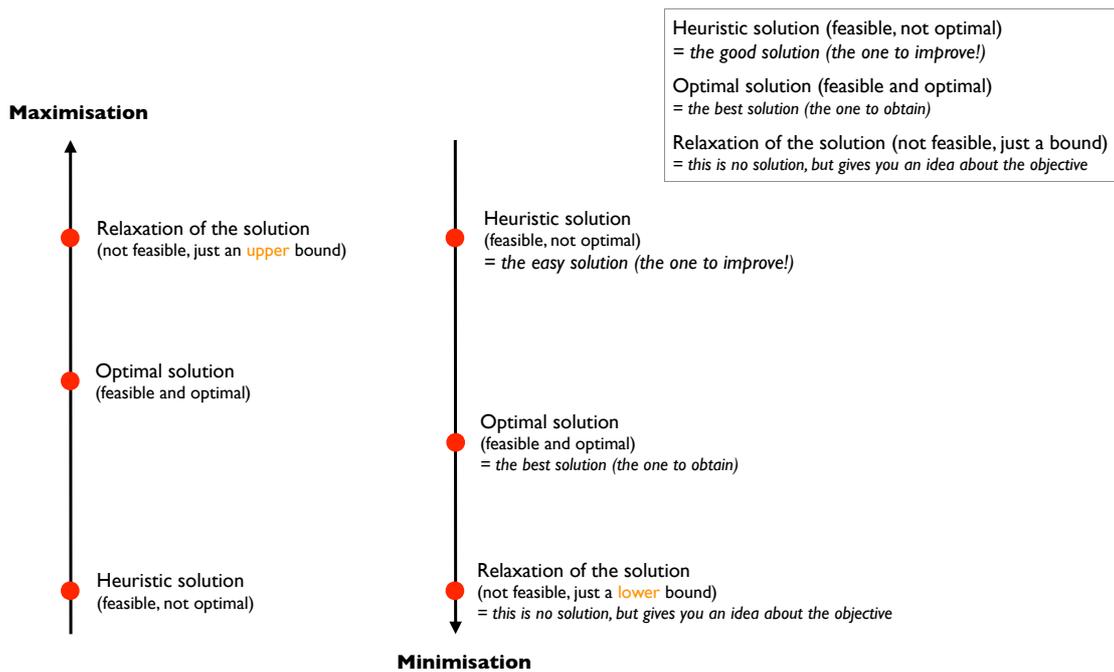
## Using bounds and relaxations

The **heuristic solution** is the solution that you obtain by solving the problem to near-optimality, or alternatively, the solution that is now currently implemented in the company. It is a feasible solution (all the constraints are satisfied) but not necessarily the best one. There might exist solutions with a better objective value. By the way, better is more when maximising the objective (e.g. profit) but less when you minimise (e.g. cost).

The **optimal solution** is unknown, unless you solve the problem to optimality with a solver and linear constraints using LP or IP, and is the solution that you want to obtain and present to the company (and if this appears to be impossible, you at least want a solution that is close to the best one, or much better than the heuristic one). The problem is that you never know how far the heuristic solution you have lies from the optimal solution you want, unless you solve the problem.

Therefore, a **relaxation of the solution** might be useful to you. It is not really a solution, since it gives you an estimate of the value of the objective which is better than the best possible one. How on earth is that possible! Well, you just relax (i.e. delete) one or more constraints, and solve the problem with a simple model or even by intuition. Often times, when deleting some constraints, the problem becomes trivial to solve, and hence, the value for the objective will be found easily. This value is optimistically good (again, good is lower for cost, and higher for profit). Think about the production of cars in a production plant with limited capacity. The sequence of cars is highly dependent on the setup times between the types of cars. Assume now that the capacity is unlimited (that is the relaxed constraint), then it's quite easy to find a production schedule, but it is not very realistic.

This principle is shown in the picture below for a maximisation problem and a minimisation problem. It is said that the relaxation of the solution is a bound, an upper bound (i.e. real solutions are worse = lower value) for maximisation problems and a lower bound (real solutions are worse = higher value) for minimisation problem.



The difference between the solution you currently have (heuristic) and the optimal solution is the maximum improvement you can ever reach, but it is unknown. The difference between the heuristic solution and the bound is an (optimistic) estimate of the improvement, and can tell you something about the potential gain. The higher, the better!

By the way, there is at least one relaxation for IP models that you already know! Recall that “relaxation” means “deleting constraints”. When you remove the integrality constraints from an IP model, you get an LP model, and hence, the solution of an LP model is a bound for the IP model. Remember the LP and IP model used before to show the size of the feasible region. The optimal LP solution had an objective value of €10, but the IP model, which is almost equal to the LP apart from one set of constraints (the integrality constraints), has a lower objective value of €9. Hence, the LP solution is an upper bound (for a maximising problem) or a lower bound (for a minimisation problem) to the IP solution.

# Stochastic models

## What is decision tree analysis?

*“No sensible decision can be made any longer without taking into account not only the world as it is, but the world as it will be.” (Isaac Asimov)*

### Decision tree analysis (DTA)

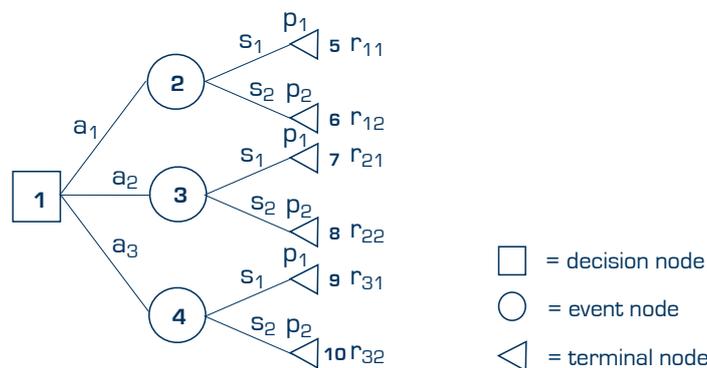
**Decision tree analysis** is a technique to choose between several courses of action in order to obtain a desired outcome within the presence of unknown events. A decision tree is a visual model consisting of nodes and arrows representing the different branches in the decision making process representing the competing alternatives the decision maker has and the possible unknown outcomes along with their respective probabilities (which sum to 1.0). The purpose of the tree is to have a visual representation of the logical sequence of decisions and uncertainties that can be used to find the best possible path that leads to the most promising reward.

### Drawing a tree

While mathematical models such as linear and integer programming models require a certain degree of expertise in mathematics, drawing a decision tree only requires a good understanding of the problem to solve, and it contains only three basic elements, as follows:

- **Actions (square):** A set of (sequential) decisions that you have to make during a period of time. The actions are within the control of the decision makers.
- **Events (circle):** Unexpected outcomes expressing possible states (i.e. scenarios) of nature, each with an estimated probability of occurrence. Events are beyond the control of the decision makers, and are ‘things’ that just happen.
- **Outcomes (triangle):** The (monetary) rewards you get if you have chosen a particular action and a certain state of the world has occurred.

Hence, drawing a tree is not very difficult, always very case specific and the central question is **“what happens next”**. It boils down to finding the logical sequence of decisions (actions) and events, and the outcomes always occur at the end of the tree. This sequence of branches containing squared nodes (decisions) and circles (possible events with probabilities) generally alternates until each path terminates in an end node (triangle) representing the award.



In the example tree, a choice must be made between a set of three actions ( $a_1$ ,  $a_2$  or  $a_3$ ), each followed by an uncertain event containing two states of nature ( $s_1$  or  $s_2$ ). These states of nature represent unknown parameter values expressed as a set of different scenarios (values) with corresponding probabilities ( $p_1$  or  $p_2$ ). The final rewards ( $r_{11}$  to  $r_{32}$ ) depend on both the action chosen by the decision maker as by the state of nature. Note that in LP and IP models, the parameter values are fixed and assumed to be known, and hence, no states of nature are incorporated in the model. Hence, the reward, which was then called the objective function value, only depends on the optimal selection of decision variables (actions) and not on the state of nature.

## Evaluating a tree

Evaluating the tree is often called **folding back the tree** as it is a gradual process working from right to left, each time reducing the size of the tree. The calculations start on the right hand side of the decision tree, and work backwards towards the left, finally ending at the root node of the tree. Each time you complete a set of calculations on a decision or event node, the remaining nodes to the right of this node that you used for the calculations that have led to the result can be removed from the tree. In doing so, you gradually fold back the tree until you end with a final value which is the solution to the problem.

Calculating the value of decision nodes is straightforward, since it represents an action within the control of the decision maker, and therefore, the best node among a set of actions (leading to e.g. the highest profit, or lowest cost) is selected. For the calculations of the values of the event nodes, a strategy must be chosen that defines the way the calculations are done. Possible strategies are:

- The average case criterion (often referred to as the expected terminal value criterion) returns the probability-weighted average of the end node values. The problem with this approach is that it may not be feasible and it is therefore the long-run average solution to the problem.
- The worst-case scenario criterion analyses the data in the tree from a pessimistic point-of-view where each value is set to its worst possible outcome, and is often useful in order to assist decision making in comparison with the average value.
- The best-case scenario criterion analyses the data in the tree from an optimistic point-of-view where each value is set to its best possible outcome, and is also helpful to assist the final decision to make. The difference between the best-case and worst-case value can be used as a measure of risk, but generally they are not always very informative and should be considered with care.
- The most likely scenario criterion aims at returning the solution with the highest probability, but as it takes only one uncertain state of nature into account it also may lead to bad decisions.

## Utility

While maximising the monetary value is often a valuable criterion for choosing the best alternative, it does not incorporate the decision maker's attitude toward risk. A decision tree assumes that each decision maker is a rational human being, and the evaluation of the tree relies on this principle, regardless of the degree of risk involved. While a rational choice might be a reasonable criterion for problems involving small investments and low degrees of risk, it is not so straightforward for very risky decisions with large capital investments.

One way to model different attitudes toward risk is to use a **utility function**. Utility theory gives a possibility to incorporate risk into decision analysis, and hence, allows the use of irrational or subjective feelings into the decision making process. Although the use of this theory might seem like a shift towards gut feeling, leaving the path of incorporating data-driven optimisation for supporting decisions, it is a strong tool to connect the hard data with the people responsible for the final go/no go decision, and hence, it brings the quantitative tools closer to their users. Utility functions are available in various ways and equations, and slightly transform the (monetary) value of the terminal nodes in utility units, to model three classes of people.

- Risk neutral: These people, if they ever exist, are just like a euro. 1 euro is 1 euro. More is more, less is less.
- Risk seeking: These people have a preference for risk. More is better, even at a higher risk.

- Risk averse: These people are afraid of risk and prefer certainty above uncertainty. More is not always better, it depends on the risk.

## Bayes' theorem

The use of decision tree analysis requires a sequential set of decisions that must be taken over time, the incorporation of unknown events and the estimate of probabilities based on some source of data (gut feeling, experience, market research, ...). The evaluation of the tree is based on these estimates (garbage in/garbage out) and returns a solution that can be chosen or not by the decision maker. However, the decision maker does not always know whether to decide immediately or delay a decision until it is clear which state of nature will occur. Hence, the important question for the decision maker is whether the waiting for additional information (to update/improve the estimates) might outweigh the decision taken now.

The **formula of Bayes** can be used to deal with dynamic and continuously updated decision making processes. Bayes recognises that decision making is often dealing with sequential events, whereby new additional information is obtained for a subsequent event, and that new information is used to revise the probability of the initial event. In the theory of Bayes, it is said that additional data can be used to update initial probabilities, known as prior probabilities, to new posterior probabilities. A prior probability is an initial probability value originally obtained before any additional information is obtained, while a posterior probability is a probability value that has been revised by using additional information that is later obtained. The formula of Bayes is then given as follows:

$$P(A | B) = P(B | A) * P(A) / P(B) \text{ or alternatively } P(B | A) = P(A | B) * P(B) / P(A)$$

with  $P(A)$  and  $P(B)$  the prior probabilities of  $A$  and  $B$  without regard to one other, and  $P(A | B)$  and  $P(B | A)$  the posterior probabilities (e.g.  $P(A | B)$  refers to the probability of event  $A$ , under the knowledge that event  $B$  has already occurred, so given that  $B$  is true).

Bayes' theorem can be the subject of the playground of Mathematics, as you can see below:

Since  $P(B | A) = P(A | B) * P(B) / P(A)$  or alternatively  $P(A) = P(A | B) * P(B) / P(B | A)$  therefore  $P(A | B) = P(B | A) * P(A) / P(B)$  and

since  $P(A | B) = P(A \text{ and } B) / P(B)$  and  $P(B | A) = P(A \text{ and } B) / P(A)$ , the following is also correct  $P(A \text{ and } B) = P(A | B) * P(B) = P(B | A) * P(A)$

And I can go on for a while...

So what? Well, you can use the formula in all kinds of ways, using simple high-school algebra, but it is a powerful tool to analyse the data you have available. If the data doesn't fit in the tree, just transform it using the formulas above, and plug it in the tree. You'll see it leads to elegant and wonderful results, and - hopefully - to better decisions.



A well-known example of the theorem of Bayes, that I simply copy pasted from the excellent book "Fooled by Randomness" by Nassim Nicholas Taleb (he copied it from the excellent Deborah Benett's book "Randomness", and nowadays you can find it everywhere on the internet) is the famous quiz given to medical doctors:

"A test of a disease presents a rate of 5% false positives. The disease strikes 1/1,000 of the population. People are tested at random, regardless of whether they are suspected of having the disease. A patient's test is positive. What is the probability of the patient being stricken with the disease?"

Most doctors answered 95%, simply taking into account the fact that the test has a 95% accuracy rate. The real answer, however, is less than 2%. Think about this. As a patient, you get a positive result and you feel miserable and afraid because of your health problems, or the problems of someone you love. But only in 2% of the cases, you (or your beloved one) actually have the disease. Think of the number of times you will be given a medication that carries damaging side effects for a given disease you were told you had, when you may only have a 2% probability of being affected with it.

The answer lies, of course, in applying the formula of Bayes.

Let A be the event that the test gives a positive result and B the event that a patient actually has the disease. Now, currently  $P(A)$  is unknown, but  $P(B) = 0.001$  and false positives  $P(A | \text{not } B) = 0.05$  and hence  $P(A | B) = 0.95$ .

We know from Bayes' theorem that  $P(B | A) = P(A | B) * P(B) / P(A)$  but since we don't know  $P(A)$  the  $P(B | A)$  cannot be calculated. Yet.

We also know that  $P(A) = P(A \text{ and } B) + P(A \text{ and not } B)$  which is, thanks to Bayes, equal to  $P(A | B) * P(B) + P(A | \text{not } B) * P(\text{not } B) = 0.95 * 0.001 + 0.05 * 0.999 = 0.0509$  or slightly more than 5%.

Now we do know  $P(A)$  and hence  $P(B | A) = P(A | B) * P(B) / P(A) = 0.95 * 0.001 / 0.0509 = 0.0186$  or almost 2%.

So why did you panic when the test turned out to be positive? Because your brain (needed for the Bayes' calculations) and your emotions (used for your reaction to the doctor upon hearing the news of the positive test) do not act in concert. We are all fooled by randomness.

**Conclusion:** Unlike linear programming and integer programming, decision trees can be easily used without a certain degree of expertise in decision modelling. It enables the user to incorporate uncertain events as long as its number is not too large. Therefore, it cannot be used if the chance event outcomes are continuous. Instead, we must redefine the outcomes so that there is a finite set of possibilities. Despite this shortcoming, it is the ideal tool to move away from your gut feeling and structure your data in a simple yet powerful tree.

# Stochastic models

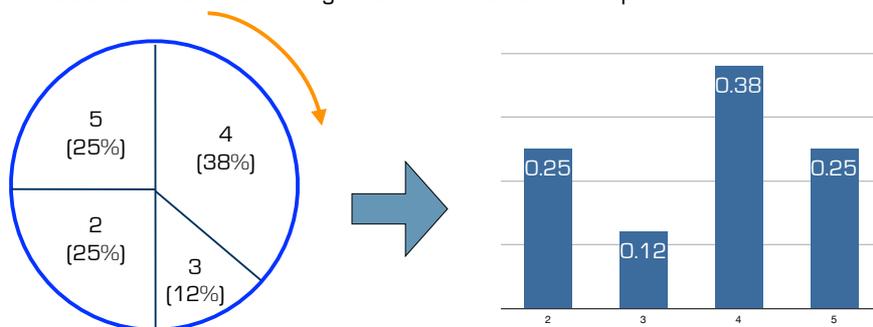
## What is Monte Carlo simulation?

*“Clever people needn't be good; they can simulate.” (Unknown)*

### Monte Carlo simulation (MCS)

**Monte Carlo simulation** is a technique that generates numbers from a predefined probability distribution to estimate the most likely outcome and the odds that certain events will occur. It repetitively produces hundreds or thousands of possible outcomes for a variable in order to measure and quantify uncertainty and chance events.

The Monte Carlo principle stems from the roulette wheels associated with the casinos of Monte Carlo that also reproduce random outcomes by turning the wheel several times. Like the wheel in the picture below containing four pieces labelled with a number, repetitively turning the wheel will reproduce the probability distribution given to the left of the figure. Consequently, reproducing this or any other probability function and generating numbers from this distribution can be easily done by constructing the roulette wheel and turning it several times on a computer.



### Statistics

Unlike a roulette wheel, the Monte Carlo method uses a standard roulette wheel that can only generate numbers between 0 and 1, with 1 not included, i.e. from the interval  $[0, 1[$ . These numbers will be used and transformed in such a way that they can be used to measure and quantify uncertainty and chance events of real systems. The wheel can be used on a computer in e.g. MS Excel using the “=rand()” command. Turning the wheel, and hence generating a new number, can be done by pressing F9. Each time a new number will be generated between 0 and 1.

So what? You don't need numbers between 0 and 1. You need numbers from a distribution you define!

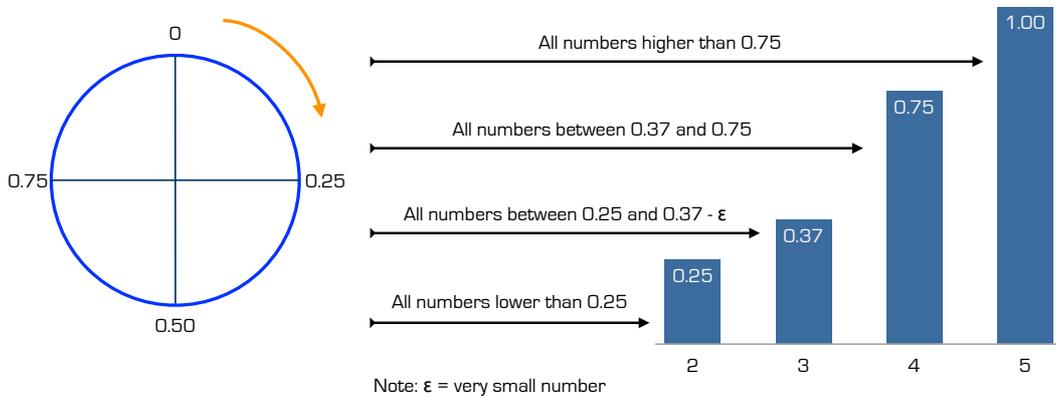
Well, that can be easily done with the same function. Consider for example the tossing of a coin, with two possible outcomes, either head or tail, with equal probability. Just print the commands below (in MS Excel) and repeat the experiment a lot of times.

```
if (rand() < 0.50) then print("Head")  
else print("Tail")
```

Or alternatively, the roulette wheel example can be easily simulated using the following commands:

```
value = rand()  
if (value < 0.25) then print("2")  
if (value < 0.37) then print("3")  
if (value < 0.75) then print("4")  
else print("5")
```

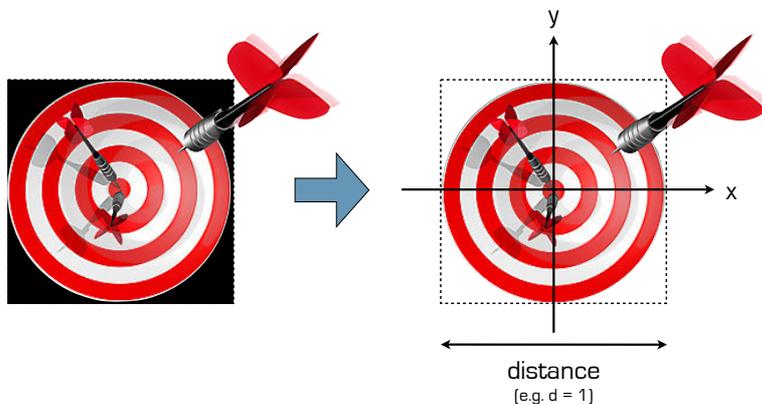
Note that the numbers 0.25, 0.37, 0.75 and 1 (for the last else-command) represent the so-called **cumulative distribution function** of the probability function of the previous figure. The wheel is used on the Y-axis of the cumulative graph, generating numbers between 0 and 1, required for any probability distribution, and finds then the corresponding numbers on the X-axis for the distribution you defined, as shown in the following picture.



While the previous distribution only generates four numbers, continuous distributions can also be easily generated. Consider e.g. the well-known normal distribution, and generate it in MS Excel using two input cells, two columns with formulas and a lot of rows (each row represents a Monte Carlo run) for the two columns. The first and second cell should be used for the mean (cell A1) and the standard deviation (cell B1). Each cell of the first column (cells C1, C2, ..., C1000) should have the “=rand()” command and the second column (in the D column) should use the MS Excel formula “=NORMINV(\$A\$1,\$B\$1,Cx)” with Cx the cell next to the cell in the D column (C1 for D1, C2 for D2, ...). Obviously, behind the “NORMINV” function, there is a formula of the cumulative distribution function of a normal distribution. No need to search it for yourself. MS Excel did it for you. If you now construct a histogram for all the numbers generated in column D, you will see a more or less beautiful normal curve. Impressing, isn't it?

## EXAMPLE

Monte Carlo simulation can be used to simulate the value of  $\pi$  ( $\pi$ ) by simulating a game of darts. Assume the circle with diagonal equal to  $d$  and a square with the same diagonal, as given in the picture below. Recall that the area of the square (S) is equal  $d^2$  and the area of the circle (C) is equal to  $\pi * (d/2)^2 = \pi * d^2 / 4$ .



Then you have the following  $C / S = \pi / 4$  or  $\pi = 4 * C / S$ .

Now you can simulate a play of darts in MS Excel, and C represents the number of times you hit and reach the circle and S the number of times you hit the square. If we then calculate  $4 * C / S$ , you have an estimated value of pi. Assume that the diagonal  $d = 1$  (but any other number can be used).

**Step 1.** Set up a column for numbering each time you throw the dart, a column to randomly generate an x-coordinate value (=rand() in cell B1) and a column to randomly generate a y-coordinate (=rand() in cell C1).

**Step 2.** Calculate the distance of the number represented by the x and y coordinates generated in step 1 from the centre of the circle. The circle on the figure is centred at the ordered point  $(x, y) = (0.5, 0.5)$  and the diagonal has a length  $d = 1$ . The equation needed to determine the distance between any two points  $(x, y)$  and  $(0.5, 0.5)$  is equal to  $\text{SQRT}[(x - 0.5)^2 + (y - 0.5)^2]$  with  $\text{SQRT}(x)$  the square root of x. So the equivalent Excel formula in cell D1 required to calculate the distance from each dart to the centre  $(0.5, 0.5)$  is, therefore

$$=\text{SQRT}[(\text{B1}-0.5)^2+(\text{C1}-0.5)^2].$$

**Step 3.** Now specify whether the point which you have generated in step 1 has hit the circle or you just missed it (and has hit the square). A “hit” occurs if the distance from point  $(0.5, 0.5)$  is less than 0.5 units, the radius of the circle, or in MS Excel in cell E1.

$$=\text{IF}(\text{D1} < 0.5, 1, 0)$$

**Step 4.** Column F will be used to calculate the percentage of hits of our darts when the experiments in steps 1 to 3 are repeated. Simply drag and drop the cells A1 to E1 to the cells below such that e.g. 100, or 500, or even more cells are filled. Assume 500 cells. The percentage of hits is then equal to the sum of all “1”s in column E divided by the number of times you repeated the experiment, i.e. 500, or in MS Excel:

$$=\text{SUM}(\text{E1:E500}) / 500.$$

**Step 5.** Recall that  $\pi$  is equal to  $4 C / S$  or 4 times the number of hits divided by the total number, or in MS Excel

$$=4 * \text{SUM}(\text{E1:E500}) / 500.$$

The value obtained will lie close to 3.1415... the real value of pi.



Download the problem (Tab “Simulate Pi”) from [www.or-as.be/books/TSBD\\_Examples.xlsx](http://www.or-as.be/books/TSBD_Examples.xlsx)

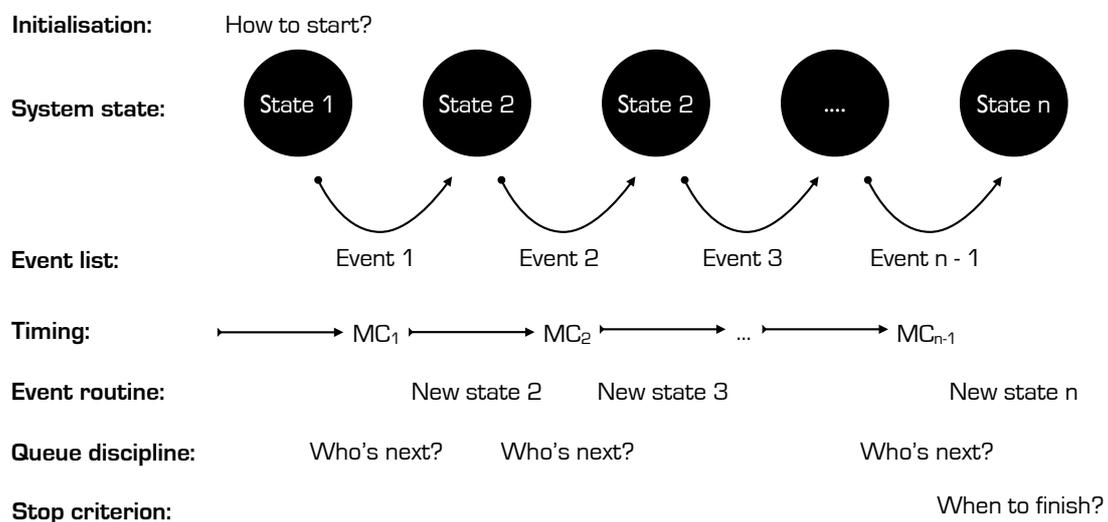
# Stochastic models

## What is discrete event simulation?

*"Reality is both continuous and discrete." (Albert Einstein)*

### Discrete event simulation (DES)

A **discrete event simulation** (DES) models the dynamics of a system as a set of discrete sequence of events in time. Each event occurs at a particular instant in time and marks a change of state in the system. An event comprises a specific change in the system's state at a specific point in time, and no change can occur between two consecutive events. An event list monitors the dynamic changes of the system and calculates the statistical behaviour upon termination of the simulation run. During the simulation run, a list of discrete events that have occurred since the process began are built up, while a second list of discrete events are pending and are assumed to occur, and this continues until the process is expected to end. Upon termination, the model collects various system statistics that measure its performance such that system bottlenecks, improvement scenarios, desired changes and more, can be easily detected.



The various components necessary to describe and model a dynamic system in a DES are given in the picture above and described along the following lines:

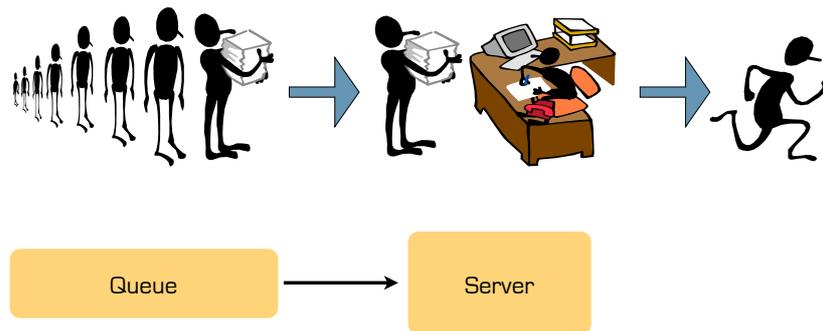
- **System state:** Description of how a particular system looks like at a particular moment in time.
- **Event list:** A list with instantaneous occurrences that changes the state of the system.
- **Timing:** A generated time moment that triggers the occurrence of an event which acts as a signal to shift to a new state (using e.g. a computer's clock variable).
- **Event routine:** A mechanism to define the updates of the system state when a particular type of event occurs (one routine for each event).
- **Queue discipline:** The way the queue in the system is organised which defines rules of inserting and removing system components (e.g. customers) to/from the queue.

Next to these five important components to set up a discrete event simulation, two additional elements need to be defined before any run can start.

- **Initialisation:** A rule that defines the system state at time 0.
- **Stop criterion:** A rule that defines when the simulation runs must end (e.g. defined as the maximum number of events, a maximum time criterion or anything else).

## Example

A single-server queuing system is the easiest system to model as a stochastic, dynamic, and discrete process and is visually represented in the picture below. The system state can be defined by the number of customers in the system and the queue size. The system follows a discrete process defined by the events, an arrival or a departure (event list) that change the system at separate points in time. The timing of the stochastic process is modelled by a Monte Carlo simulation that generates random variables to inter-arrival times and service times (timing). The way customers are removed from the queue and put to the server follows a first-in first-out mechanism (queue discipline). When customers arrive, they either join the queue or in case no one is waiting and no one is being served, they go immediately to the server (event routine). When customers leave the system, the new state depends on the size of the queue (when it is empty, the system is temporary idle until a new customer enters the system) (event routine).



The main executive routine to run the simulation on a computer consists of the following commands:

- Step 1.** Set clock = 0
- Step 2.** Set cumulative statistics = 0 (e.g. average waiting times, # customers waiting, ...)
- Step 3.** Define initial system state (e.g. queue empty, server idle)
- Step 4.** Generate the occurrence time of the first arrival and put it in the event list
- Step 5.** Select the next event on event list (arrival or departure)
- Step 6.** Advance simulation clock to time of next event (Monte Carlo simulation)
- Step 7.** Define the new state with this event (using this event's event routine)
- Step 8.** If stop criterion not met, goto step 5

## Software

While Monte Carlo simulation can be easily done in MS Excel, modelling a discrete event simulation is not so straightforward and requires dedicated open source or commercial software tools. Obviously, the use of these tools requires some expertise and involves a learning curve, but generally most tools are not very difficult in use. Simple systems can however be modelled in MS Excel, such as the examples described in the well-known book *Factory Physics* written by Wallace Hopp and Mark Spearman who introduce a framework for manufacturing management.

	Interarrival time $AT_i$	Arrival $A_i$	Waiting time $W_i$	Processing time $PT_i$	Total time $TT_i$	Departure $D_i$
<b>Row 1. State 1</b>	<i>Monte Carlo!</i>	$A_1 = A_{i1} + AT_1$	$W_1 = \max(0, D_{i1} - A_1)$	<i>Monte Carlo!</i>	$TT_1 = W_1 + PT_1$	$D_1 = A_1 + TT_1$
<b>Row 2. State 2</b>		$A_2 = A_{i1} + AT_1$	$W_2 = \max(0, D_{i1} - A_2)$	<i>Monte Carlo!</i>	$TT_2 = W_2 + PT_2$	$D_2 = A_2 + TT_2$
<b>Row 3. State 3</b>		$A_3 = A_{i1} + AT_1$	$W_3 = \max(0, D_{i1} - A_3)$	<i>Monte Carlo!</i>	$TT_3 = W_3 + PT_3$	$D_3 = A_3 + TT_3$

Discrete events occur along the rows!  
(for use in MS Excel)

The system that is modelled represents a flow shop of three machines with waiting times (representing inventory) and service times (representing production times). The production process follows a logical sequence from raw materials to machine 1, then to machine 2 and finally to machine 3 and the product is ready to be shipped. Each product is a single entity in the system and is modelled by a single row in the simulation. The discrete events, that model additional products to be produced, are modelled by the different rows. Hence, the discrete events are modelled along the different rows in MS Excel, hereby simulating a dynamic system over time.

In the MS Excel file, the stop criterion is set to the production of 2,000 items, and the system performance is measured by the average waiting and service times, the variability in waiting times, and these calculations are done for each of the three machines as well as for the overall system performance. Initial probability distributions for modelling the arrival times and service times for each machine, to be used by the Monte Carlo simulation, are set as uniform distributions with minimum and maximum values that can be changed by the user. A demo version of this simulation can be downloaded from the link given below.



Download the problem (Tab "Factory Physics") from [www.or-as.be/books/TSBD\\_Examples.xlsx](http://www.or-as.be/books/TSBD_Examples.xlsx)

# Warehousing

Optimising every little facet of your warehouse



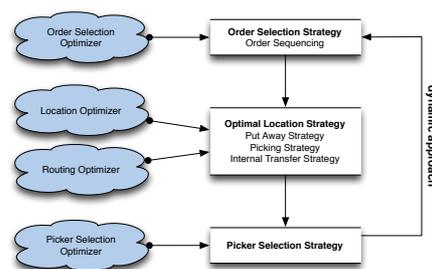
## Problem Description

Warehouse optimisation helps companies to save money on their internal supply chain in order to improve their own logistics by increasing productivity in picking, optimising placement of stock and shaping batches of optimised orders.

The vast majority of existing software tools focuses on the collection of data to construct a quick and easy warehousing schedule. Through the use of a set of simple and easy standard optimisation rules, they mainly focus on a fast construction of warehouse schedules. However, due to their often weak optimisation algorithms, their focus is restricted to improving standard costs often leading to small or even insignificant

improvements. The black box of the underlying algorithms often does not allow to fully integrate the software in the current best practices of the company.

Experiments illustrate the importance of simulation experiments and the use of advanced optimisation techniques instead of a simple standard set of rules used in current software tools.



## Design

The simulation/optimisation algorithm makes use of seven optimisation engines:

**Put Away Strategy.** Determine the best locations to put away orders taking into account the capacity constraints.

**Picking Strategy.** Determine the best locations to pick orders taking into account the inventory availability.

**Batching Strategy.** Determine the optimal timing of a set of orders for one external client (truck).

**Stock/Buffer Transfer Strategy.** Generate automatic policies to optimise the put away strategy for external warehouses.

**Internal Transfer Strategy.** Generate automatic internal transfer orders to optimise the warehouse system.

**Zoning Strategy.** Determine the relevant set of criteria to split up a warehouse in zones.

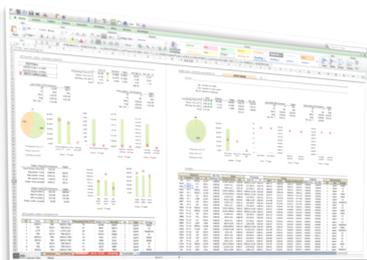
**Routing Strategy.** Optimise the routing of orders in a warehouse based on vehicle routing optimisation algorithms.



## Solution Approach

The integrated optimisation model is a tailor-made optimisation system that is easy to maintain to construct a fully optimised warehouse scheduler.

The development of a set of well-considered and tested optimisation rules based on the company data has resulted in a fully optimised system in C++. This system can be used to easily carry out a set of simulation experiments searching for the best performing [better, not only faster] and robust [against an ever-changing situation] optimisation rules validated on the company data.



For more information, visit [www.optimisedwarehousing.com](http://www.optimisedwarehousing.com).

# Nurse rostering

Providing high-quality care to patients



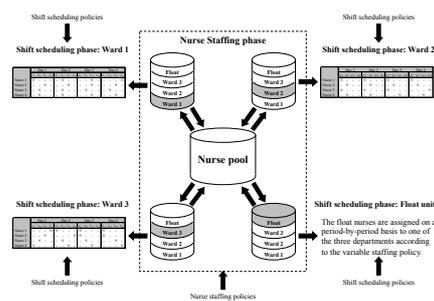
## Problem Description

The efficient and effective management of nursing personnel is of critical importance in a hospital's environment comprising approximately 25 % of the hospital's operational costs. The nurse organisational structure and processes highly affect the nurses' working conditions and the provided quality of care.

The impact of different nurse organisation structures and different organisational processes is investigated for Belgian university hospitals using optimisation models. In order to make accurate nurse staffing decisions, the employed solution methodology incorporates shift scheduling characteristics in

order to overcome the deficiencies of the many software models.

Using a methodology that integrates the elements of the staffing and scheduling decision phase, different perspectives and new trends to patient oriented structures in the healthcare sector have been evaluated.



## Design

The personnel unit head formulated various [conflicting] goals that represent the hospital's policy and the nurses' preferences. The quality of a nurse staffing and shift scheduling plan is measured in three dimensions as follows:

**Effectiveness.** Providing high-quality nursing care is reflected in the continuity of nursing care, the presence of a core nursing staff and the absence of frequent schedule changes.

**Efficiency.** The hospital management aims to ensure a cost-efficient service of a nursing unit establishing a close match between the expected patient demand with the staffing size and available nursing skills.

**Job satisfaction.** The quality of a nurse roster is measured in terms of personnel job satisfaction which is related to the working atmosphere, the fairness among nurses, the compliance with nurses' wishes and requests and the healthiness of nursing schedules.



## Solution Approach

A new methodology is developed that incorporates the shift scheduling phase in the nurse staffing decision in order to determine an optimal nurse staffing and shift scheduling plan simultaneously.



The optimisation procedure is a column generation procedure (branch-and-price) that splits the

problem in two subproblems. The master problem is a multi-objective generalised set partitioning problem that can be formulated as a linear integer programming model to appoint a nurse directly to a certain duty roster and a specific department. The individual schedules are defined by the subproblem that aims to construct a feasible roster line with respect to all time-related constraints for one single employee.



"Analyzing the nursing organizational structure and process from a scheduling perspective", Health Care Management Science, 16,177-196, 2013.

# Steel production

## Multi-objective finite-capacity production scheduling

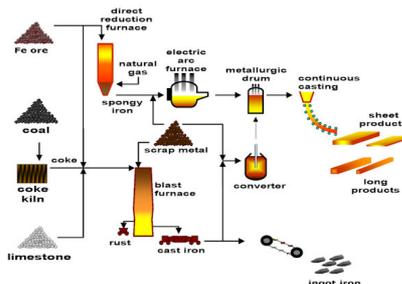


### Problem Description

A multi-objective finite-capacity production scheduling algorithm at a middle-term planning horizon level (between a long-term aggregate planning and a short-term machine scheduling level) was developed for an integrated steel company in Belgium. Annual production is 5 million tons of flat steel strip for the automotive industry and for all kinds of high-quality applications such as domestic appliances, sanitary products, heating, construction and furniture. The company handles every step of the production process from the supply of raw materials to the coating of steel and the production of laser-welded blanks.

The disaggregate planning tier constructs a schedule that assigns all operations for all orders taking various constraints and multiple objectives into account

Four types of constraints have been identified and multiple objective functions have been used to minimise the total cost as a sum of five different cost functions.



### Design

A finite capacity planning model that includes a three-tier hierarchical production planning process is developed, containing an aggregate planning, a disaggregate planning and a sequencing phase. The disaggregate planning phase consist of three modules:

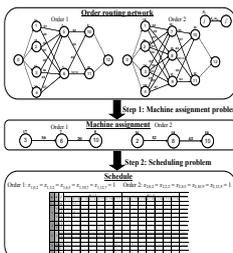
**Routing optimisation.** The routing of an individual order is determined through the network of production steps.

**Machine assignment.** Each order is assigned to one machine for each production step of its routing (assignment constraint), resulting in the total assignment cost

**Scheduling.** All operations of each order need to be scheduled on a particular day, taking capacity constraints, precedence constraints and setup constraints into account, as well as multiple objectives (lateness costs, earliness costs, utilisation costs and production flow costs).



### Solution Approach



The algorithm consists of two solution steps. A machine assignment step assigns each order to a unique machine for each production step. The second step constructs a schedule where each operation of all orders is assigned to a

particular day, given the assigned machines of the previous step. To determine which orders should be selected for scheduling at each machine during each day, knapsack problems were constructed that take capacity constraints, precedence constraints and setup constraints as well as multiple objectives (lateness costs, earliness costs, utilisation costs and flow costs) into account.



"A finite-capacity production scheduling procedure for a Belgian steel company", International Journal of Production Research, 47, 561 - 584, 2009.

# Job shop scheduling

Industrial wheels and rubber castors manufacturing



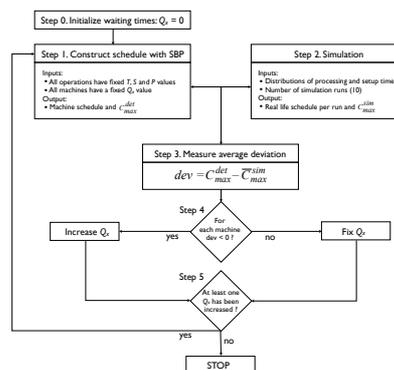
## Problem Description

The manufacturing process for the production of industrial wheels and rubber castors is optimised. The company provides customised products for all areas of the industry, including the forklift industry, the glass-industry, the roller coaster industry and the automotive industry.

Due to the diversity and large amount of orders, the production process can be considered as a very complex environment, with thousands of orders, each having an often slightly different routing across the five work centres. With the presence of sequence-dependent setup times, parallel machines and assembly structures, the production process

has a strong similarity with the complex job shop scheduling problem.

While the current planning approach was based on standardised rules-of-thumb and experience, the use of an optimised solution approach could possible lead to huge benefits to the company.



## Design

The scheduling model incorporates various case-specific elements, such as:

**Setup times.** The order and machine specific setup times are based on the company's historical data.

**Group technology.** When setup times are sequence-dependent, combining orders with similar tool requirements on a particular machine can be advantageous.

**Transportation times.** The transportation times between machines are estimated based on historical data and include both the real transportation time and the waiting times before transportation.

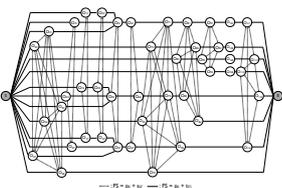
**Unlimited capacity.** Some of the machines have an unlimited capacity such that this production step can be considered as a transportation time rather than as a processing time.

**Assembly and split structures.** Different parts (the rim, flange and hub) need to be assembled, each requiring a certain pre-processing that needs to be incorporated in the model.



## Solution Approach

Since their current way of working contains a well-considered mix of machine bottleneck detecting, order



priority setting to level workload and a simplified system to cope with variability and unexpected events, it is not surprising that the general idea of the shifting bottleneck

procedure sounded an attractive alternative to improve their current way of working.

The procedure is an extension of a hybrid shifting bottleneck procedure with a tabu search algorithm while incorporating various company specific constraints. Simulation experiments have tested the impact of group technologies and the alignment of waiting times to machine criticality.



"Applying a hybrid job shop procedure to a Belgian manufacturing company producing industrial wheels and castors in rubber", Computers and Industrial Engineering, 61, 697-708, 2011.

# Airline crew scheduling

Drastically reduce operational costs of airline companies



## Problem Description

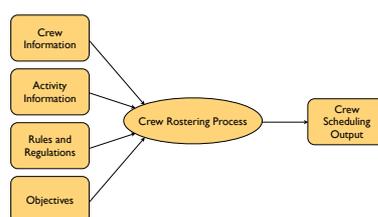
The crew scheduling problem in the airline industry is extensively investigated in the operations research literature since efficient crew employment can drastically reduce operational costs of airline companies.

Given the flight schedule of an airline company, crew scheduling is the process of assigning all necessary crew members in such a way that the airline is able to operate all its flights, and constructing a roster line for each employee minimising the corresponding overall cost for personnel.

Depending on the workload variability and the company's policy

to support long-term schedule transparency, an airline company can select to construct a pure cyclical approach, an 'ad hoc' approach with fixed building components or a mixture of both approaches.

The analysis of the best approach leading to an overall cost reduction is a task typically done using a computerised optimisation approach.



## Design

The analysis is done using a modular design consisting of the 6 modules described along the following lines:

**Module 1.** The demand modelling step to identify the required duties to deliver service.

**Module 2.** Determination of how rest days are to be combined with working days.

**Module 3.** Shift scheduling to generate duties and assignment of the number of employees to each duty.

**Module 4.** Line of work construction to create timetables for each staff member over the rostering horizon.

**Module 5.** Allocation of one or more tasks to particular lines of work associated with specific staff skills or levels of seniority.

**Module 6.** Assignment of individual staff to the constructed lines of work.



## Solution Approach

A scatter search algorithm with solutions obtained by an exact branch-and-price procedure and a steepest

descent variable neighbourhood search is used to generate feasible but not necessarily optimal solutions. The objective function minimises the cost of regular, extra, and freelance

personnel but also takes crew fairness and preferences into account.

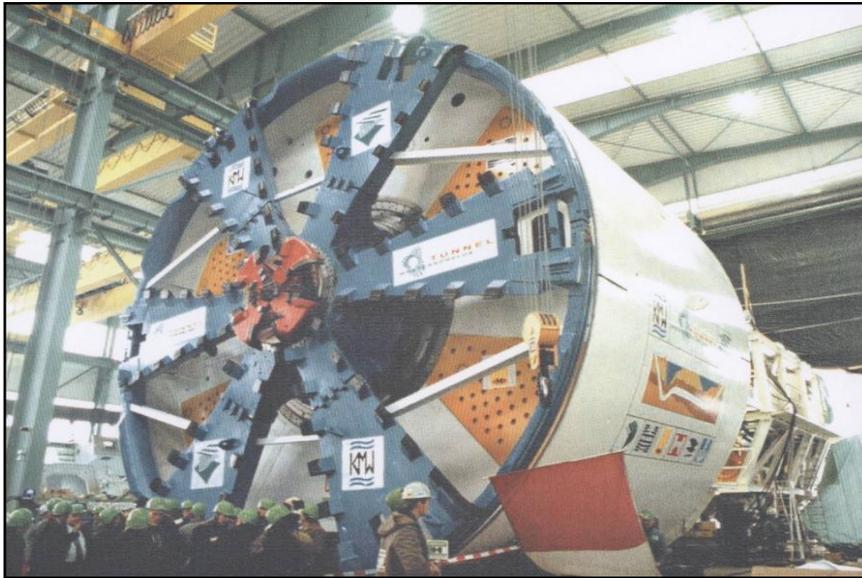
The constraints incorporate various crew personal records (e.g. hours flown, trainings) and qualifications (e.g. seniority, list of destinations, language proficiency, skill competency) as well as the company specific and European rules and regulations concerning flight times, rest periods, etc.



"A hybrid scatter search heuristic for personalized crew rostering in the airline industry", European Journal of Operational Research, 206, 155-167, 2010.

# Tunnel construction

Scheduling repetitive activities for a huge project

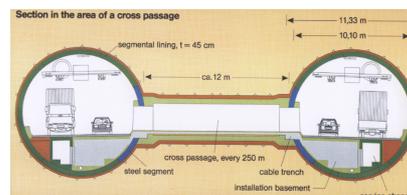


## Problem Description

The construction of the Westerscheldetunnel was a huge project with a groundbreaking boring technique at the Netherlands. This tunnel provides a fixed link between Zeeuwsch-Vlaanderen and Zuid-Beveland, both situated in the Netherlands with a length of 6.6 kilometres. There are two tunnel tubes and in each tube, there are two road lanes.

The total project (the main tunnel, 22 kilometres service roads, several entry roads and viaducts, toll square, etc...) took 5 years to complete. The construction started at the end of 1997 and the completion date was 14 March 2003 (on my birthday!), with a cost of 750 million euro. The

construction was a technically unique project. Most tunnels in Europe are built in hard, rocky material. Never before in Western Europe has a tunnel so long or so deep been bored through relatively soft substrates such as sand and clay. The deepest point lies 60 metres below sea level. The construction logistics were extremely complex because many different actions had to take place simultaneously and so they had to be very well planned.



## Design

The design of the model takes three important aspects into account.

### Work continuity constraints.

The application of crew work continuity constraints provides room for an effective resource utilisation strategy by minimising crew idle time. This leads to the maximisation of the benefits from the learning curve effect and the minimisation of the off-on movement of crews on a project once work has begun.

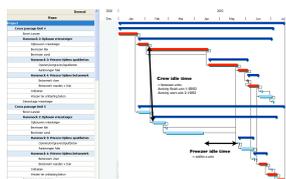
**Project duration.** Minimising the project duration is a more complex process for repetitive projects than for non-repetitive ones. The simple logic of crashing critical activities does no longer hold with the presence of work continuity constraints.

**Repetitive work.** Most construction projects contain both repetitive and non-repetitive activities. While the non-repetitive activities can be scheduled using traditional network techniques, the repetitive activities require more specialised tools.



## Solution Approach

The algorithm is a **three-phased horizon-varying approach** in order to find the complete trade-off profile between resource idle time and total project duration.



The three steps consist of an activity labelling step to schedule activities using the same resource close to each

other in time, the construction of a search tree to model the repeating character of the activities and a recursive search to optimise the work continuity of the repetitive project.

The work continuity objective guarantees that the idle times of important capital intensive resources are minimised, and therefore, contributes to the overall cost minimisation objective of the model.



"Work continuity optimization for the Westerscheldetunnel project in the Netherlands", Tijdschrift voor Economie en Management, 52, 435-449, 2007.

# Machine scheduling

Optimising knitted fabric production processes



## Problem Description

A case specific machine scheduling model was developed for a Belgian textile manufacturer using a production line which produces knitted fabric for mattresses. The circular knitting machines used to produce the fabric are the bottleneck in this production process, and are therefore the focus of this study. These machines operate in a parallel layout, where multiple machines produce simultaneously.

The manufacturer has a large product portfolio, and the differences in production times between orders can be significant. This is further complicated by the fact that the knitting machines used at the manufacturer are not all of the same

make. The production infrastructure consists of machines of different brands as well as different ages.

Prior to the start of a knitting job the machine settings have to be adjusted. This includes emptying the machine, adjusting the yarns and adjusting the setting of the ring on which the needles are positioned. The length of this setup procedure is strongly dependent on the preceding job. Elements which influence the length of the changeover are the number of yarns which have to be replaced and the direction (up vs. down) of the ring adjustment.

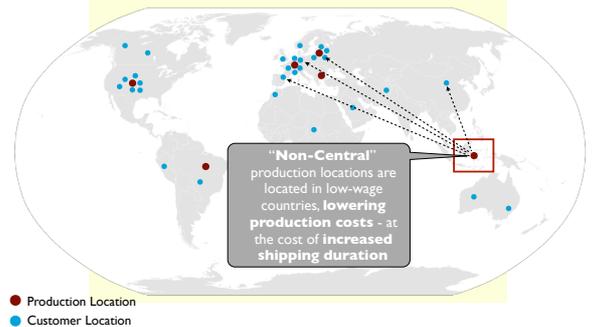
## Design

The design takes 3 very specific components into account which makes the problem hard to solve with traditional machine scheduling tools.

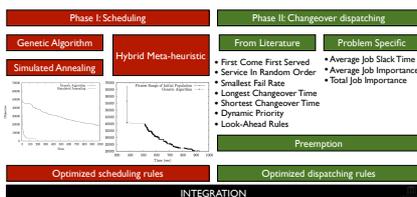
**Geographically dispersed production locations.** The production is not limited to the Belgian production facilities, but is strongly geographically dispersed, and is located both at locations with relatively high wage costs (USA, Western Europe and the Baltic States) and locations in low-wage countries (Indonesia, Brazil and Eastern Europe).

**Unrelated parallel machines.** The production times of a product depend on the machine to which it is assigned, as well as the product type and quantity which has to be produced.

**Sequence dependent changeover times.** Each machine's setup time or cost for a particular job is determined not only by that job but also by the previous job that the



## Solution Approach



The algorithm consists of a two-phased solution approach and includes a hybrid simulated

annealing and the genetic algorithm as well as various dispatching rules.

The first phase encompasses the scheduling problem, including the geographically dispersed production locations, but omitting the changeover interference problem. The output of this first stage is then used as the input for the second phase, which mitigates the effects of the changeover interference using dispatching rules.



"Scheduling of unrelated parallel machines with limited server availability on multiple production locations: a case study in knitted fabrics", International Journal of Production Research, 52, 2630-2653, 2014.

# Course assignment

Enabling individual satisfaction and group fairness



A mathematical model formulation for the assignment of cases to students to be solved in a group assignment is used in a class environment.

Using a simple graphical user interface allows a simple translation of the decision variables, constraint definition, and objective formulation in understandable concepts. The model incorporates the preferences of the students (input data) and is used in a

class discussion. A solution is shown at the end of the discussion in which students are assigned to groups and case studies.

The exam cases are selected from different disciplines, such as production, finance, and human resource management. This makes it more likely that the student's preferences are distributed equally over the cases, which leads to a

## Design

The model is built during a class discussion and differs along the expertise and background of the student population, but they often incorporate the following components.

**Satisfaction.** Maximising the total group preferences is the single objective to satisfy the students.

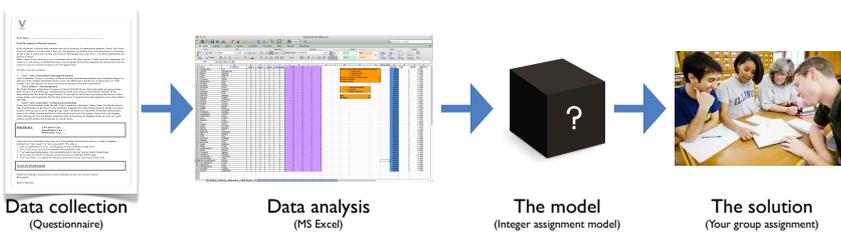
**Assignment.** Each student is assigned to a single group to solve exactly one case.

**Group size.** The maximum size of the group is set based on the complexity of the case studies.

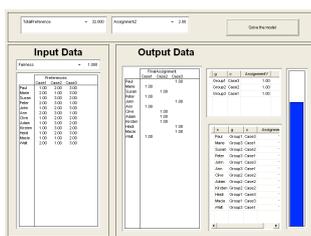
**Fairness.** The different case studies are equally distributed among the groups.

**Background.** A constraint ensures that the advanced modelling students, if any, are equally divided among the groups.

**And more.** Additional constraints such as gender equilibria, individual preferences for teams, and much more, can be easily included.



## Solution Approach



An integer programming model is built and solved in a software tool AIMMS. The problem can be modelled in various ways, and decisions have to be made for the choice of the decision variables (two-stage or

three-stage variables) and the uses of advanced concepts such as big-M constraints.

To further reduce the complexity, students need to become aware of alternative problem formulations. The simplifications can often be reached by a clever understanding of the problem that must be solved rather than investigating the technical details of the model.



"Introducing optimization techniques to students: An exam case distribution model", INFORMS Transactions on Education, 10, 53-61, 2010.

# Custom pack design

grouping of medical disposable surgery items



## Problem Description

A model for the design of custom pack to be used in the Operations Room of a Belgian hospital was developed. The introduction of custom packs into the operating theatre is beneficial for the clinical quality of care. Since custom packs directly reduce material handling effort, they decrease the risk that nurses contaminate sterile products. Less unwrapping of medical items is required, which results in a more stable air flow within the operating room and less impact of dust particles. Custom packs also allow for a quick response to material intensive emergency services due to speeding in surgery setup. Yet, the most important driver for using custom

packs is the gain in operational efficiency and the resulting cost savings

Custom packs are effective in reducing the time needed for surgery setup. The items are no longer individually packaged so that unwrapping the custom packs replaces the individual unwrapping of all medical items that are included. Moreover, medical items do not have to be sorted anymore since they are organised and sequenced within the custom pack, irrespective of the nurse who is in charge of this process.

## Design

The design of the model relies on a two-phase heuristic that builds upon the strengths of mathematical programming without engaging into advanced modelling techniques.

**Heuristic pack generation.** A heuristic generates in a first phase a set of custom packs that might prove useful in minimising the points of touch.

**Optimal pack selection.** A mixed-integer linear programming model is defined which selects the best combination of custom packs defined in the first phase, while keeping the flexibility to still configure some custom packs completely from scratch during optimisation.

This dual generation/selection approach gives an opportunity for the algorithm to discover a custom pack configuration which we might have missed during the first phase and would contribute to the overall solution. As we only incrementally allow for additional packs in the second phase, we limit the number of variables and hence simplify the search process.



## Solution Approach



The study presents a mathematical programming approach to guide hospitals in developing or reconfiguring their custom packs. It aims at minimising points of

touch, which we define as a measure for physical contact between staff and medical materials. It starts from an integer non-linear programming model, but was translated into an exact linear programming (LP) solution approach and an LP-based heuristic. Next, a simulated annealing approach is developed to benchmark the mathematical programming methods.



"On the design of custom packs: grouping of medical disposable items for surgeries", International Journal of Production Research, 53, 7343-7359, 2015.

# Research



# Optimisation in project scheduling

## Exact and heuristic methods



### PUBLICATIONS

(selection)

1

#### A decomposition-based genetic algorithm for the resource-constrained project scheduling problem

Operations Research, 55, 457-469, 2007

2

#### A hybrid scatter search/electromagnetism meta-heuristic for project scheduling

European Journal of Operational Research, 169, 638-653, 2006

3

#### An evaluation of the adequacy of project network generators with systematically sampled networks

European Journal of Operational Research, 187, 511-524, 2008

4

#### The impact of various activity assumptions on the lead time and resource utilization of resource-constrained projects

Computers and Industrial Engineering, 54, 140-154, 2008

5

#### Download a complete publication list from [www.or-as.be/grants](http://www.or-as.be/grants)

Ghent University and the Operations Research & Scheduling group present:

## Resource-constrained project scheduling algorithms

The **resource-constrained project scheduling problem** (RCPSP) is the basic problem type in project scheduling and aims at the minimisation of the total project duration. It is one of the most widely studied project scheduling problems in literature, and it has resulted in an overwhelming amount of papers with solution procedures to solve the problem.

### Algorithms

#### Solve project instances

In the early research endeavours to solve resource-constrained project scheduling problems, exact techniques such as IP formulations or branch-and-bound methods were used to solve small project instances to optimality, while often using extensive computer time.

Nowadays, problem instances have grown both in size and increased in complexity and have resulted in a need for fast and efficient meta-heuristic search algorithms such as genetic algorithms, scatter search methods, tabu search procedures and similar techniques that are able to heuristically solve large problem instances in reasonable time.

In this research study, new hybridised solution approaches have been developed that consist of a well-designed mix of the best features of existing algorithms. They have resulted in algorithms such as a hybrid version of the "Electromagnetism" and "Scatter Search" techniques (cf. publication 2).

Furthermore, a decomposition based algorithm has been developed that outperforms the state-of-the-art algorithms from literature, and iteratively focuses on subparts of the project to locally optimise these subproblems, aiming at reaching an improved global solution.

Extensions to more practical settings to test the impact of realistic activity features on the utilisation of the project resources has been investigated in detail (cf. publication 4).

### Data

#### Test the new procedures

In the RCPSP research community, a lot of comparisons have been made, mostly done on the **PSPLIB** data instances generated by ProGen. In our paper published in Operations Research in 2007 (cf. publication 1), a new dataset RG300 has been created by the project data generator RanGen (cf. publication 3) which can be freely downloaded from the **OR & S** website ([www.projectmanagement.ugent.be](http://www.projectmanagement.ugent.be)). Both the new dataset and the data generator are publicly available to stimulate future research in this interesting and challenging research domain.



# Optimisation in healthcare

## Scheduling nurses in hospitals



### PUBLICATIONS

(selection)

1

#### **An Electromagnetism meta-heuristic for the nurse scheduling problem**

Journal of Heuristics, 13, 359-385, 2007

2

#### **Comparison and hybridization of crossover operators for the nurse scheduling problem**

Annals of Operations Research, 159, 333-353, 2008

3

#### **On the characterisation and generation of nurse scheduling problem instances**

European Journal of Operational Research, 196, 457-467, 2009

4

#### **A genetic algorithm for the nurse rostering problem**

Computers & Operations Research, 38, 1400-1411, 2011

5

#### **Download a complete publication list from**

[www.or-as.be/grants](http://www.or-as.be/grants)

Ghent University and the Operations Research & Scheduling group present:

## Algorithms for the nurse scheduling problem

**Personnel scheduling** problems are encountered in many application areas, such as public services, call centres, hospitals, and industry in general. For most of these organisations, the ability to have suitably qualified staff on duty at the right time is of critical importance when attempting to satisfy their customers' requirements and is frequently a large determinant of service organisation efficiency. This explains the broad attention given in literature to a great variety of personnel rostering applications.

### Personnel scheduling

#### Using algorithms for people

In general, personnel scheduling is the process of constructing occupation timetables for staff to meet a time-dependent demand for different services while encountering specific workplace agreements and attempting to satisfy individual work preferences. The particular characteristics of different industries result in quite diverse rostering models which leads to the application of very different solution techniques to solve these models. Typically, personnel scheduling problems are highly constrained and complex optimisation problems.

### Scheduling nurses

#### Meta-heuristic optimisation of nurse shifts

The **nurse scheduling problem** (NSP) is a well-known scheduling problem which assigns nurses to shifts per day taking

both hard and soft constraints into account. The objective is to maximise the preferences of the nurses and to minimise the total penalty cost from violations of the soft constraints. In this research track, we present various novel meta-heuristic techniques based on the principles of electromagnetism [cf. publication 1], genetic algorithms [cf. publication 4] and scatter search methods. Furthermore, a benchmark of the building blocks with other existing methods is made and published in publication 2.

### Data

#### Test the new procedures

In order to test the new procedures, a data generator NSPGen and a corresponding dataset NSPLib with benchmark instances have been developed [cf. publication 3]. All best known solutions, the problem generator as well as executables for various meta-heuristic procedures to solve the nurse scheduling problem can be downloaded from the OR&S website.

([www.projectmanagement.ugent.be](http://www.projectmanagement.ugent.be)).



# Optimisation in project scheduling

## Multi-mode trade-offs for project activities



### PUBLICATIONS

(selection)

1

#### A genetic algorithm for the preemptive and non-preemptive multi-mode resource-constrained project scheduling problem

European Journal of Operational Research, 201, 409-418, 2010

2

#### Using resource scarceness characteristics to solve the multi-mode resource-constrained project scheduling problem

Journal of Heuristics, 17, 705-728, 2011

3

#### An artificial immune system algorithm for the resource availability cost problem

Flexible Services and Manufacturing, 25, 122-144, 2013

4

#### An experimental investigation of metaheuristics for the multi-mode resource-constrained project scheduling problem on new dataset instances

European Journal of Operational Research, 235, 62-72, 2014

5

#### Download a complete publication list from

[www.or-as.be/grants](http://www.or-as.be/grants)

Ghent University and the Operations Research & Scheduling group present:

## Multiple activity modes in project scheduling

The **resource-constrained project scheduling problem with multiple modes** (MRCPSP) is an extension of the well-known single-mode RCPSP and involves the selection of a time/resource combination for each activity such that the total project duration is minimised.

The time/resource trade-off involves the selection of an activity duration and corresponding resource use, and a shorter duration is linked to a higher resource use. Obviously, the periodic use of resources is restricted to a predefined maximum, and can therefore never be exceeded.

### Algorithms

#### Solve project instances

The increasing interest in operations research for meta-heuristics during the recent years has resulted in the development of several meta-heuristic solution procedures for the MRCPSP. A wide variety of meta-heuristic strategies, solution representations and schedule generation schemes were used to develop the most efficient algorithms.

Several heuristic optimisation algorithms have been developed to solve the MRCPSP, such as a genetic algorithm with a fast and efficient local search procedure and the ability of coping with activity splitting (cf. publication 1), an artificial immune system approach (cf. publication 3) and a scatter search procedure that dynamically selects the local search methods based on resource specific information (cf. publication 2).

Moreover, in a summary paper (cf. publication 4), an overview is given of the existing meta-heuristic solution procedures to solve the MRCPSP. All algorithms have been coded in C++ and a fair comparison is made between the different meta-heuristic algorithms on the existing benchmark datasets and on a newly generated dataset.

Computational results are provided and recommendations for future research are formulated.

### Data

#### Test the new procedures

Despite the availability of project data instances in the literature to test new algorithms, all available sets show some shortcomings given the recent evolution in the development of meta-heuristic solution procedures. It is for this reason that three new datasets for the multi-mode scheduling problem are proposed that overcome these shortcomings. These sets, known as MMLIB50, MMLIB100 and MMLIB+ can be used to benchmark future algorithms to solve this challenging scheduling problem.

The new data instances can be downloaded from the OR&S website ([www.projectmanagement.ugent.be](http://www.projectmanagement.ugent.be)).



# Optimisation in production scheduling

## Scheduling jobs on machines



### PUBLICATIONS

(selection)

1

**Applying a hybrid job shop procedure to a Belgian manufacturing company producing industrial wheels and castors in rubber**

Computers and Industrial Engineering, 61, 697-708, 2011

2

**A hybrid single and dual population search procedure for the job shop scheduling problem**

European Journal of Operational Research, 215, 512-523, 2011

3

**A hybrid genetic algorithm for the single machine maximum lateness problem with release times and family setups**

Computers and Operations Research, 39, 2346-2358, 2012

4

**A comparison of priority rules for the job shop scheduling problem under different flow time- and tardiness-related objective functions**

International Journal of Production Research, 50, 4255-4270, 2012

5

**Download a complete publication list from**

[www.or-as.be/grants](http://www.or-as.be/grants)

Ghent University and the Operations Research & Scheduling group present:

## Algorithms for machine and job shop scheduling problems

The scheduling of production systems is a widely investigated branch of the operational research domain. Machine scheduling refers to problems in a manufacturing environment where jobs have to be scheduled for processing on one or more machines to optimise one or more objectives. Within the machine scheduling field there is a large variety of problem types, based on the characteristics of the jobs, the restrictions of the process and the objectives to be optimised. Both the **machine scheduling problem** and the **job shop scheduling problem** have been solved using various meta-heuristic solution procedures.

### Algorithms

#### Solve artificial problems

Several machine scheduling problems, have been investigated at the OR&S group, ranging from the single machine environment to the multi-stage job shop environment (i.e. single and parallel machine scheduling problems, traditional and flexible job shop scheduling problems, etc...), with varying job characteristics, process restrictions and objective functions.

For these problems, various algorithmic optimisation approaches were developed, such as an electromagnetism algorithm hybridised with tabu search optimisation, as well as a single population genetic algorithm and a dual population scatter search procedure to solve the well-known job shop scheduling problem [cf. publication 2].

The current state-of-the-art research results to solve the single machine scheduling problem with release times, due dates and a maximum lateness objective have been summarised in a research paper and book chapter. Moreover, this single machine maximum lateness problem with release times was extended with family setup times [cf. publication 3] and with precedence constraints between jobs.

A comparison and validation of various priority rules for the job shop scheduling problem under different objective functions was made [cf. publication 4]. Several priority rules from literature are used to schedule job shop problems under two flow time-related and three tardiness-related objectives.



### Validation

#### Solve real problems

Although most algorithms have been tested on artificial data instances, available at the OR&S research website [www.projectmanagement.ugent.be](http://www.projectmanagement.ugent.be), some algorithms have been adapted and fine-tuned to the settings of company specific problems. They have resulted in a procedure to solve a complex variation of the parallel machine scheduling problem, based on a case study at a Belgian manufacturer of knitted textiles, as well as in a new procedure to solve a job shop scheduling problem at a Belgian manufacturer producing industrial wheels and castors in rubber [cf. publication 1].

# Optimisation & logic

## Scheduling problems with SAT solvers



### PUBLICATIONS

(selection)

1

**An evaluation of the adequacy of project network generators with systematically sampled networks**

European Journal of Operational Research, 187, 511-524, 2008

2

**Multi-mode resource-constrained project scheduling using RCPSP and SAT solvers**

European Journal of Operational Research, 213, 73-82, 2011

3

**Hybrid tabu search and a truncated branch-and-bound for the unrelated parallel machine scheduling problem**

Computers and Operations Research, 53, 107-117, 2015

4

**A new solution approach to solve the resource-constrained project scheduling problem with logical constraints**

European Journal of Operational Research, 2, 577-591, 2016

5

**Download a complete publication list from [www.or-as.be/grants](http://www.or-as.be/grants)**

Ghent University, University of Lisbon and the OR&S group present:

## Using the Boolean Satisfiability Problem for scheduling problems

The **Boolean Satisfiability Problem**, abbreviated as SAT, is the problem of determining the existence of an interpretation that satisfies a given Boolean formula. The set of logical equations and solution procedures to solve these challenging problems originates from the Computer Science world, and has been adapted to be used in project scheduling in a research collaboration between Ghent University (Belgium), Universidade Aberta (Portugal), and the laboratory of Artificial Intelligence and Decision Support of the Institute for Systems and Computer Engineering - Technology and Science (INESC TEC, Portugal).

### SAT Algorithms

#### Satisfying constraints in scheduling

Project scheduling is the act of determining a timetable for project activities under a set of constraints, including precedence relations, scarce resource and case-specific needs of the project. Due to the complexity of these constraints, finding a feasible and/or optimal schedule is often hard. In this project, a SAT search is used to satisfy a diverse set of constraints, and this algorithm is embedded in a meta-heuristic search to optimise the construction of the schedule once a feasible SAT solution is found.

Our efforts have led to an algorithm for solving the so-called "multi-mode resource-constrained project scheduling" (cf. publication 2). Moreover, extensions to logical constraints, including OR and

bi-directional relations between activities, have resulted in the SAT based solution approach of publication 4.

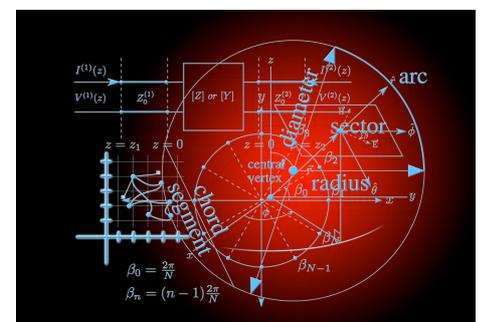
An extension to machine scheduling (cf. publication 3) with a tabu search algorithm is also published.

### Data

#### Test the new procedures

Since the use of SAT algorithms to schedule projects with additional constraints is relatively new in the academic literature, most project data that are currently available cannot be used for this research project. Therefore, the currently existing datasets have been extended to incorporate the specific features and constraints used in the scheduling algorithms. To that purpose, the RanGen generator has been used (cf. publication 1) for which more information is available at the OR&S website.

[\[www.projectmanagement.ugent.be\]](http://www.projectmanagement.ugent.be)



# Statistical project control

## Advanced statistics for Earned Value Management



### PUBLICATIONS

(selection)

1

#### Setting tolerance limits for statistical project control using earned value management

Omega - International Journal of Management Science, 49, 107-122, 2014

2

#### A comparison of the performance of various project control methods using earned value management systems

Expert Systems with Applications, 42, 3159-3175, 2015

3

#### Developing a framework for statistical process control approaches in project management

International Journal of Project Management, 33, 1289-1300, 2015

4

#### A multivariate approach to statistical project control using earned value management

Decision Support Systems, 79, 65-76, 2015

5

#### Download a complete publication list from

[www.or-as.be/grants](http://www.or-as.be/grants)

Ghent University and the Operations Research & Scheduling group present:

## Statistical project control using Earned Value Management

Project control systems must indicate the direction of change in preliminary planning variables compared with actual performance. In case the project performance of projects in progress deviates from the expected planned performance, a warning must be indicated by the control system as a trigger to take corrective actions.

### Statistical project control Triggers for actions

A new **Statistical Project Control (SPC)** approach based on the principle of statistical process control charts is presented in order to improve the discriminative power between normal and abnormal project progress situations. Based on the existing and commonly known Earned Value Management (EVM) metrics, the project control charts will have an improved ability to trigger actions when variation in a project's progress exceeds certain predefined thresholds. These control charts make use of tolerance limits obtained from simulations (cf. publication 1) or of more advanced multivariate statistics (cf. publication 4). Moreover, the newly developed methods are compared and benchmarked with existing methods in literature (cf. publications 2 and 3).

### Methodology

#### Simulating project progress

A large number of simulation experiments has been set up running on Ghent University's **super computer**

**infrastructure**, leading to gigabytes of data in order to test the ability of the statistical project control charts to discriminate between random and assignable variation. An intensive analysis of the generated data is done to compare the use of statistical project control limits with traditional earned value management thresholds and to validate their power to report warning signals when projects run into danger.

### Experiments

#### Analysing terabytes of data

The results of the computational experiments show that:

- The use of SPC outperforms the best practices in EVM.
- The Earned Schedule (ES) approach performs better than the traditional EVM approach.
- A combined use of X-charts and XR-charts allows to detect a variety of project problems.
- An extended multivariate analysis control approach leads to control efficiency improvements.

All experiments have been carried out by generating project data and artificially simulating project progress using P2 Engine ([www.p2engine.com](http://www.p2engine.com)) and a thorough analysis of the terabytes of data in R ([www.r-project.org](http://www.r-project.org)). More information and free downloads are available on the OR&S website ([www.projectmanagement.ugent.be](http://www.projectmanagement.ugent.be)).



# Controlling projects

Time/cost trade-offs and artificial intelligence



## PUBLICATIONS

(selection)

1

### Support Vector Machine regression for project control forecasting

Automation in Construction, 47, 92-106, 2014

2

### A study of the stability of Earned Value Management forecasting

Journal of Construction Engineering and Management, 141, 1-10, 2015

3

### A comparative study of artificial intelligence techniques for project control forecasting

Expert Systems with Applications, 46, 249-261, 2016

4

### A study on complexity and uncertainty perception and solution strategies for the time/cost trade-off problem

Project Management Journal, 47, 29-50, 2016

5

### Download a complete publication list from

[www.or-as.be/grants](http://www.or-as.be/grants)

Ghent University and the Operations Research & Scheduling group present:

## Artificial intelligence for project scheduling and control

One of the key aspects in controlling projects is the ability to provide an accurate and reliable forecast for the expected duration and cost of the project in progress. Reliable estimates on these expected values should facilitate the corrective actions that must be made when projects tend to run out of control. Often, trade-offs between time and cost objectives must be made, and various alternative ways of solving project problems is therefore a matter of degree.

### Artificial intelligence

#### Improving forecasting accuracy and stability

The data overload provided by the periodic Earned Value Management (EVM) metrics during project control allows the project manager to forecast the expected final time and cost of the project in various ways. Through the use of advanced artificial intelligence methods such as support vector machines (cf. publication 1), decision trees, bagging, random forests and boosting (cf. publication 3), both the stability and the accuracy of these forecasts can be dramatically improved (cf. publication 2), hereby leading to better and more reliable decisions made by the project manager.

### Time/cost trade-offs

#### Taking corrective actions

One of the most obvious and widely applied decision techniques that can be used to bring projects in danger back on

track is the so-called crashing method. In this method, additional resources are added to activities in delay, such that their expected duration decreases, while their cost rises. The way these time/cost trade-off decisions are made depends on the uncertainty perception of the project manager as well as on the underlying complexity of the project (cf. publication 4).

### Classroom experiments

#### Simulating project progress

The classroom is the ideal environment to artificially test the behaviour of people (project managers) in a controlled setting. Two games, known as the Project Scheduling Game (PSG) and its follow-up edition PSG Extended are available to simulate various classroom experiments on time/cost project control. While PSG focuses on the dynamic progress of projects and the time/cost decision that must be made to deliver the project on time at the lowest possible cost, PSG Extended focuses on the underlying solution approach/mechanism taken by participants who have successfully played the Project Scheduling Game. More information on these games is available at the PM Game Center website ([www.pmgamecenter.com](http://www.pmgamecenter.com)).



# Projects contracting

## Incentives for time, cost and scope



### PUBLICATIONS

(selection)

1

#### Incentive contract design for projects: The owner's perspective

Omega - The International Journal of Management Science, 62, 93-114, 2016

2

#### Extensions of earned value management: Using the earned incentive metric to improve signal quality

International Journal of Project Management, 35, 148-168, 2017

3

#### A parallel multi-objective scatter search for optimising incentive contract design in projects

European Journal of Operational Research, 261(3), 1066-1084, 2017

4

#### Multi-mode schedule optimisation for incentivised projects

Under submission

5

#### Download a complete publication list from

[www.or-as.be/grants](http://www.or-as.be/grants)

Ghent University and the Operations Research & Scheduling group present:

## A research study on incentivised project management contracting and control

Incentive contracts are often used in project management to align the conflicting objectives of project owners and the contractors they employ to execute a project. Whereas project owners have great interest in the outcome of the project with respect to duration, cost and scope, the contractor often remains solely interested in his short term profit. Especially when contracts are linked to multiple performance dimensions, optimising the design and execution of such contracts becomes nontrivial. While the project owner is mainly interested in maximising the utility derived from the project's outcome, the contractor's main concern is to optimise the inherent risks and trade-offs encountered in the specific project contract.

### Incentive contract design

#### The Owner's perspective

The research study aims to unify the largely qualitative body of literature on contract design with more quantitative research from the area of project management. This is done through the creation of novel quantitative models which are able to accurately describe incentive contract structures as well as relevant project properties. The study provides prescriptive insights on the impact of incentive contracts for the different areas of project management. It provides guidelines for the design of a project contract that includes incentives from the owner's perspective (cf. publication 1).

The models developed in this study allow the use of advanced optimisation methods from other research domains such as project scheduling and control (cf. publications 3 and 4). Hence, the study also provides guidelines on how can incentivised projects be scheduled in order to maximise profit?

### Controlling incentives

#### The Earned Value Management system for project incentives

Incentivised contracts should be monitored and controlled during the project progress. Therefore, the research has also shown that the earned-incentive control system is able to control the incentives negotiated between contractors and owners of the project, and therefore, can be considered as a valuable extension of the earned value methodology.

### Experiments

#### Validating the incentivised methods

Both artificial data from literature as well as empirical data collected from international companies are used to represent a wide range of possible incentive contracting situations.



# Project optimisation

## Payment models and net present value



### PUBLICATIONS

(selection)

1

**A new scheduling technique for the resource-constrained project scheduling problem with discounted cash flows**

International Journal of Production Research, 53, 2771-2786 , 2015

2

**Payment models and net present value optimization for resource-constrained project scheduling**

Computers and Industrial Engineering, 91, 139-153 , 2016

3

**Capital- and resource-constrained project scheduling with net present value optimization**

European Journal of Operational Research, 256, 757-776 , 2017

4

**Metaheuristics for the discrete time/cost trade-off problem with net present value optimization and different payment models**

Under submission

5

**Download a complete publication list from**  
[www.or-as.be/grants](http://www.or-as.be/grants)

Ghent University and the Operations Research & Scheduling group present:

## Heuristic algorithms for payment models in project scheduling

Optimizing the Net Present Value (NPV) of a project is a research topic that finds its roots in a paper written in 1970. Rather than focusing solely on the project duration minimisation, the inclusion of the time value of money using the NPV concept leads to better and more realistic schedules. A project's NPV however depends on the timing and amount of payments, and hence on the timing of the baseline schedule. In literature, a number of payment models exist ranging from payments occurring once an activity has been completed, to payments at predetermined points in time. Depending on the selected payment model, different schedules lead to different NPVs. Consequently, each payment model requires a different baseline schedule to fully optimise the project's NPV.

### Payment models

#### Resource and time/cost optimisation

Resource-constrained project scheduling with discounted cash flows is a topic that has been investigated since decades. In this study, activity cash flows are optimised from the point of view of the contractor (cf. publication 1). The goal is to design an appropriate meta-heuristic, and compare existing payment models. The proposed algorithm has operators (e.g. local search) fine-tuned for each payment pattern to clearly demonstrate the differences between the models, and their influence on the resulting schedules. The research has been

extended to alternative payment models (cf. publication 2), capital constraints (cf. publication 3) as well as time/cost trade-offs (cf. publication 4).

### Methodology

#### Metaheuristic search

The employed methodology focuses on integrating the NPV maximisation objective within the resource-constrained scheduling algorithms. A genetic algorithm (GA) is used as method to further improve generated project schedules by combining them to construct better solutions. A GA is one of several well-known evolutionary algorithms based on organic evolution, and makes use of crossover operators to combine several existing schedules, mutation to change a single schedule, and selection to determine what schedules are maintained.

### Experiments

#### Simulating project progress

Computer experiments show that payment models have a strong influence on the timing of the baseline schedule, and that adding payment models to time/cost optimisation can further increase the realism of cash flow models in project scheduling.



# Controlling projects

Exponential smoothing/reference class forecasting



## PUBLICATIONS

(selection)

1

### Construction and evaluation framework for a real-life project database

International Journal of Project Management, 33, 697-710, 2015

2

### Empirical evaluation of earned value management forecasting accuracy for time and cost

Journal of Construction Engineering and Management, 141, 1-13, 2015

3

### Practical application and empirical evaluation of reference class forecasting for project management

Project Management Journal, 47, 36-51, 2016

4

### Improving project forecast accuracy by integrating earned value management with exponential smoothing and reference class forecasting

International Journal of Project Management, 35, 28-43, 2017

5

### Download a complete publication list from [www.or-as.be/grants](http://www.or-as.be/grants)

Ghent University and the Operations Research & Scheduling group present:

## Empirical evaluation of project forecasting and control approaches

Project forecasting and control is crucial for the success of a project, and researchers have therefore spent huge amounts of time on developing and improving predictive techniques. Most of these techniques have been tested in academic settings, on artificial project data. Until now...

### Project data

#### Construction and evaluation of empirical database

For many years, both researchers and practitioners have expressed the need for a large and diverse real-life project database, based on which a wide range of project management techniques could be empirically validated and evaluated. Since the well-known DoD database (the American Department of Defense) is not accessible to foreign researchers and professionals, such a database has been created in this study, following a standardised quality-assuring approach.

The database currently consists of +100 projects from various sectors and sizes, with a large portion of them containing actual progress data, and is continuously growing. The database construction procedure was supported by the introduction and application of so-called **project cards**, a standardised tool for project data categorisation, evaluation and acquisition (cf. publication 1). A project card provides information about a project's characteristics regarding baseline schedule, activity risk and project control. Based on these project

cards, the quality of a project's baseline data and tracking data can be assessed, as well as gaps in the initial database (regarding sector, size, authenticity, etc.) can be identified and then filled via a targeted search for appropriate data.

### Forecasting

#### Time and cost forecasting

The database has been used for testing the accuracy of several forecasting methods, such as Earned Value/Earned Schedule forecasts for time and costs (cf. publication 2) as well as exponential smoothing techniques (cf. publication 3).

### Reference classes

#### Simulating project progress

Reference class forecasting or comparison class forecasting is a method of predicting the future by looking at similar past situations and their outcomes (cf. publication 4). The theories behind reference class forecasting were developed by Nobel prize winner Daniel Kahneman (cf. other article in this book) and Amos Tversky. In the study, it has been shown that reference class forecasting techniques can significantly improve the accuracy of time and cost predictions for projects.



# What's your story?

## Share content and get involved



*“Any fact becomes important when it's connected to another.” (Umberto Eco)*

### Connecting ...

In an age where social media are used everywhere and connecting the world is an ambition of all of us, I wonder whether there is also a place for this desire for connection in the academic world. I'm still not sure, but I think there is. Although I was initially sceptical against all that connected stuff, I gradually learned that connecting things and people doesn't do any harm to anyone. Instead, it gives you a way to share your work with people who want to use it, enhance it and criticise it. Besides, research is not made to stay in the ivory tower. It's there to be spread, to be picked up by someone who wants to elaborate on the idea, to be criticised and to be enhanced. The ivory tower of researchers is now open to the world, via many, many, probably too many connections.

### ... research and researchers

Various ways of connecting researchers exist, and there are probably more than described here. A list of three well-known ways for connecting researchers is given below:

- **ORCID** (Open Researcher and Contributor ID) is a nonproprietary alphanumeric code to uniquely identify scientific and other academic authors. It provides a persistent digital identifier that distinguishes every researcher and, through integration in key research workflows such as manuscript and grant submission, supports automated linkages between the researcher and his or her professional activities ensuring that the work is recognised. More information can be found at [www.orcid.org](http://www.orcid.org) (ID = 0000-0001-6702-3563).
- **ResearcherID** is a similar identifying system for scientific authors, introduced by Thomson Reuters. It also acts as a central place to manage and share professional information of researchers, and can be accessed at [www.researcherid.com](http://www.researcherid.com) (ID = D-8647-2015).
- **Research Gate** is a social networking site for scientists and researchers to share papers, ask and answer questions, and find collaborators. It is the Facebook for researchers, and although I am not very active on it, it sometimes helps me in finding sources in literature, or connecting to peers. It can be accessed on [www.researchgate.net](http://www.researchgate.net).

### ... students and professionals

- **ORASTalks** is the reference to connect people with an interest in the Operation Research methodology and its application in Project Management to the research and professional activities of OR-AS (company) and OR&S (research group). The connection can be made via the free app ([www.or-as.be/orastalks](http://www.or-as.be/orastalks)), via twitter (@ORASTalks) or by joining the Facebook group page ([www.facebook.com/groups/ORASTalks](http://www.facebook.com/groups/ORASTalks)).
- **OR-AS blog** is a communication channel to connect people with an interest in the professional and research activities of OR-AS ([www.or-as.be/blog](http://www.or-as.be/blog)). Its periodic updates are also available via the ORASTalks app.
- **LinkedIn**, world's largest professional network, is also occasionally used to share the content on Decision Making and Project Management ([www.linkedin.com](http://www.linkedin.com)).
- **Twitter** shares short and easy-to-read messages with links to the blogs mentioned above. Next to the @ORASTalks profile, they can also be read from the personal twitter profile via @MarioVanhoucke.

# Education

# Introduction

Operations Management: Improving business decisions for undergraduates, graduates, postgraduates and MBA students



*“Education is the movement from darkness to light.” (Allan Bloom)*

## Operations Management

While this book focuses rather on the methodology (Operations Research) than on the application of this methodology for business, the different programmes at universities and business schools aim at combining both the methodology and the application in an integrated Operations Management approach. Operations Management is an area of management concerned with overseeing, designing, and controlling the process of production and redesigning business operations in the production of goods or services. It involves the responsibility of ensuring that business operations are efficient in terms of using as few resources as needed, and effective in terms of meeting customer requirements. It is concerned with managing the process that converts inputs (raw materials, labour, and energy) into outputs (goods and/or services). In the literature, this discipline is called Operations Management, or otherwise referred to in this book as decision making for business.

In the next pages, the Operations Management major of the Business Engineering programme at Ghent University is discussed in detail, as well as the course outlines of the different modules and programmes at Ghent University, Vlerick Business School and UCL School of Management.

- Master in Operations Management (Ghent University):
  - Do cool things that matter.
  - It isn't big data but big algorithms that is going to change the world.
  - The movie: Out now!
- Course outlines:
  - Integrated Management Exercise (B.Sc. programme).
  - Decision Making for Business I, also known as Decision Sciences, or Taking Sound Business Decisions (MGM and MBA programmes).
  - Decision Making for Business II, formerly known as Applied Operations Research (M.Sc. programme).
  - Project Management, also known as Dynamic Project Planning (M.Sc. and MGM programmes).

While much of the material is focused on the Business Engineering - Operations Management programme at Ghent University, parts of the course modules are also used for other programmes, as will be discussed below.

## Institutions, programmes and course modules

Below, a short description is given of the programmes where the course modules are given. Most of the presentation of the programmes is a simple copy-paste of the institutions' website, since I had no idea how to describe the programmes better than what was given there.

The **Business Engineering - Operations Management programme** at the Faculty of Economics and Business Administration of Ghent University has started in 2004 (B.Sc.) and 2007 (M.Sc.) and was loosely based on the M.Sc. Operations and Technology Management. Ever since its introduction, it immediately attracted hundreds of students with an interest in business, management and technology. In both the B.Sc. and M.Sc. programmes, different management science principles are discussed and maintained throughout the courses including mathematical modelling, statistics and numerical algorithms to improve an organisation's ability to enact rational and meaningful management decisions. With a strong foundation in business economics, a broad knowledge of new technologies and a strong focus on quantitative analytics in production, services, logistics, marketing and finance, business engineers understand better than anyone else how to improve the efficiency in the various parts of a company's logistic chain ([www.ugent.be/eb/en](http://www.ugent.be/eb/en)).

The **Full-time MBA** at Vlerick Business School is an intimate cohort of ambitious and collegial individuals who come to Vlerick with a diversity of ideas, backgrounds and aspirations. The programme is structured in such a way that each individual is guaranteed exposure to all facets of business management knowledge. It also enables each individual to tailor the experience to his or her individual needs. This is accomplished through the delivery of a wide range of elective modules, the choice of a learning project and an international trip ([www.vlerick.com](http://www.vlerick.com)).

The **Master in General Management (MGM)** programme at Vlerick Business School is an intensive programme to give the young student the solid grounding he/she needs in all core management domains to maximise the career opportunities. Whatever the student's study background or future ambitions, the programme teaches the necessary management skills to develop true potential. Half-way through the programme the student can tailor his/her curriculum through a broad offer of 3 specialisation tracks, 10 hands-on boot camps, and by choosing a company project or global social project.

The UCL School of Management offers undergraduate, graduate and executive programmes in management, technology entrepreneurship, information management for business, management science and business analytics. The programmes incorporate learning activities and industrial experiences that will help develop commercial competencies, internal organisational management skills, and a high degree of creativity, preparing scientists, engineers, physicians, and other innovators for leadership roles in the next generation of technology-intensive organisations, as well as in finance and consulting firms. The **M.Sc. Management Science Programme** is designed for students who aim at a career at the world's leading companies that need people who can operate in complex, innovation-intensive, data-driven environments. The programme aims at delivering people who can analyse problems using quantitative tools and qualitative methods, take decisions in the face of uncertainty and risk, and deliver results through people. Management Science provides a rigorous, practical foundation in these critical skills ([www.msi.ucl.ac.uk](http://www.msi.ucl.ac.uk)).





# Master in Operations Management

It isn't big data but big algorithms that is going to change the world.

*"Change is the end result of all true learning." (Leo Buscaglia)*

## Operations Management

**Operations Management** is an area of management concerned with overseeing, designing, and controlling the process of production and redesigning business operations in the production of goods or services. It involves the responsibility of ensuring that business operations are **efficient** in terms of using as few resources as needed, and **effective** in terms of meeting customer requirements. It is concerned with managing the process that converts inputs (raw materials, labour, and energy) into outputs (goods and/or services).

The **Master in Operations Management** of the Business Engineering programme will focus on the most relevant topics of Operations Management, which is often referred to in the literature as **Management Science** or **Business Analytics** or in business as **Supply Chain Management** or **Logistics**. The two-year Master will highlight various business aspects of Operations Management in a learning-by-doing environment and will put forward a well-balanced combination of theoretical lectures, practical business games and case studies, as well as guest lectures and company projects.

The important business concepts within Operations Management will be highlighted from various angles and perspectives. A stream centred around **supply chain concepts** such as lean management, agile management, risk management and cost/benefit management will illustrate how various concepts from the Bachelor years are applied in a real-world business decision making setting.

A stream centred around **business ICT** with attention to ICT and business alignment of **project management**, including project management best practices such as Earned Value Management (EVM) and Schedule Risk Analysis (SRA) will showcase best practices in ICT and project management based on experience in world leading companies. Within this stream, a lot of attention will be spent to recent evolutions within the business software decision making tools, with a focus on the recent evolutions on business analytics and big data.

Concepts such as e-business, six-sigma quality management, supply chain mapping, third party logistics as well as supply chain innovations such as fully automated warehousing, advanced inventory management, database management in decision making, and much more will be discussed, not only from a theoretical point of view but almost always illustrated from practical experience and with business cases.

Teaching is not a static process but rather requires a continuous dynamic update to new relevant business topics. Therefore, many of the topics are given in **collaboration with companies** involved in the programme. The member companies of PMI Belgium and EVM Europe, but also others such as Arcelor Mittal, Fabricom GDF Suez and CERN (the European organisation for nuclear research) often play an active role: They define relevant topics, follow the work done by students and sometimes even award the most relevant outcomes with a recognition (cash prize, award, ...).

## Courses

### Master 1

**Project Management** provides an understanding of key issues and applied methodologies relating to Integrated Project Management and Control. It provides the essentials a project manager should have when faced with preparing the work necessary for managing and controlling projects in progress, with a clear focus on integrating scheduling, risk and control to set up a project management and control system using the available tools and techniques and best practices.

**Production Strategy.** In order to achieve flexible, cost-effective production systems required to survive in today's volatile global markets, a thorough understanding of the basic dynamics of factories/services and their link with the competitive strategy of a firm is essential.

**Total Quality Management** deals with questions on how to produce high quality goods or services, discusses ways to discover the reliability of suppliers, presents techniques on how to decide whether or not to accept the raw materials received, assesses software tools to monitor production systems with high quality standards, and much more.

### Master 2

**Decision Making for Business (or Applied Operations Research)** is an integrated decision making case study in which students are responsible for a real business problem. They should act as a consultant and must analyse and define the company problem, suggest a novel solution, think about an implementation strategy as well as validate the return of investment for their suggested approach.

**Supply Chain Management.** The objective of this course is to make the students aware of the importance of supply chain management and logistics and to teach them the concepts and techniques necessary to analyse and optimise a supply chain. This way companies are in a position to respond to the increasing pressure to shorten delivery times, enhance flexibility and reduce costs.

## Blended learning

All courses are given in a flexible and dynamic teaching environment rather than in a traditional ex cathedra teaching method. This style and method of teaching is known as **blended learning**, and includes the use of case studies, business games, software tools, and all other digital techniques available in the classroom. It is an ideal way to design courses that mix different kinds of teaching methods and supportive material to engage students and bring them closer to the relevance of the course content, hereby stimulating engagement, involvement and even enthusiasm resulting in a better learning experience.

## Who

The Master in Operations Management is targeting young **undergraduate students** with a background and experience in production management and information science. All topics are relevant for a professional career in both the private and public sector, and apply to supply chain managers, data scientists, marketeers, project managers, financial engineers, human resource managers, and many others who are responsible for business processes with critical performance, time and budget targets.

## Lecturers

All lecturers have academic and professional experience within their specific domain and will highlight the course themes from both an academic point of view as from a business relevance perspective, hereby giving novel themes and recent developments a central place in their course curriculum.

**Mario Vanhoucke** has +15 years of experience in Project Management and Decision Making for Business with teaching assignments at universities in Belgium, the UK and Suriname, business schools in Belgium, Lithuania and China and for various companies. His experience from collaborations with member companies from PMI Belgium and EVM Europe are used as case studies throughout his teaching sessions.

**Tarik Aouam** has gained his experience in Production Planning, Risk Management and Supply Chain Integration as a lecturer in universities in Morocco, the United Arab Emirates and the US. His experience as senior analyst in various international companies is used throughout his Supply Chain Management lectures.

**Dries Goossens** has gained his experience in Combinatorial Auctions, Procurement and Optimisation in Transportation Problems as a lecturer in Belgium, and occasional stays in the Netherlands and Finland. His involvement in the design of a combinatorial auction for Solids, in cooperation with Housing Association Stadgenoot, is used in his teaching.

## Guest lecturers

The **Department of Business Informatics and Operations Management** combines the lectures of our core professors with guest lectures given by nationally and internationally recognised leaders in the field of Operations Management, active at renowned companies such as Möbius, Delaware Consulting, GEO Intelligence, PWC, Groenewout, Cordys, Accenture and Solventure.

## Why

Today, companies must make **better** and **faster** decisions about their customers, competitors, partners, and operations by turning tons of **data** into valuable business information. Creating successful business strategies must be accomplished by cleverly combining huge amounts of **information**, skills of different **people** and knowledge of new **technologies** into a single business intelligence system. However, since the managerial landscape is defined by situations of **risk**, **uncertainty** and **continuous changes**, these business intelligence systems should be put into the right perspective, and care must be taken about their use and relevance in practice. **Business intelligence** systems should not be considered as automatic decision making systems, but rather as **decision support** systems that help to make decisions for business problems which may be rapidly changing and not easily specified in advance. These systems can be either fully computerised or might require human input, but are preferable a combination of both, such that they **support** and **facilitate** the decision making process and lead to **improved business solutions**.

Everybody's talking about "big data", but few are talking about how to get **from data-rich to decision-smart**. In this master, it will be shown how to get from data discovery to return on investment and real business value and how to bridge the gap between decision makers, IT managers and analytics professionals. Welcome to the expedition from business problem to analytics solution. Welcome to the exciting world of **Operations Management!**

# Master in Operations Management

The movie: Out now!



Learn what the Master in Operations Management at the Faculty of Economics and Business Administration of Ghent University has to offer!



**Master in Operations Management**  
It isn't **big data** but **big algorithms** that is going to **change the world**.

*"Change is the end result of all true learning." (Leo Buscaglia)*

slides brochure flyer app art

QUESTION

WHY TO CHOOSE THE  
MASTER IN OPERATIONS MANAGEMENT?

WATCH THIS MOVIE TO GET THE ANSWER

This promotion video can also be viewed on YouTube

Watch this short movie on YouTube (<https://youtu.be/U-t9JBIG7zc>)

# Course outline

## Integrated Management Exercise



Data driven decision making to finance your startup company's growth

*"A good decision is based on knowledge and not on numbers." (Plato)*

### Integrated Decision Making exercise

This data driven decision making course module to finance a company's growth was developed at the faculty of Economics and Business Administration of Ghent University and serves as an introduction to decision making for Bachelor students in Business Engineering. The students have to solve an integrated case study in a limited period ( $\pm 5$  days), hereby learning to translate results of models into practical solutions and guidelines. The purpose of these case studies is to emphasise the integration between the management of the different functional domains on the operational level. Moreover, the course module gives students the opportunity to apply the techniques and concepts already taught in different courses into the context of a real company. The students will work in small groups in order to thoroughly analyse a case study, and report their findings in a paper and present their results to a team of experts. It is the intention that the students use knowledge and abilities acquired during the Bachelor programme to conduct independent and scientific (hence critical) research. In this way they will acquire the skill to analyse and solve problems in a scientific manner. At the same time students will have to collaborate with different teams of students and will need to write down and present their findings appropriately and convincingly. This enhances the students' communicative skills and their ability to work in team [more info is available at [www.pmgamecenter.com](http://www.pmgamecenter.com)].

#### Day 1.

**Introduction to the case study.** In the first lecture, a short introduction to the case study and brief explanation of expectations for the next 5 days are given. The students are responsible for a startup company that is currently in its 3<sup>rd</sup> year. The company can no longer finance its growth and is in urgent need for money. The assignment of the 1<sup>st</sup> day mainly consists of reading the case study, analysing its data and setting up a company strategy that defines all actions of the future assignments.

#### Day 2.

**Demand forecasting.** In a 2 to 3 hours lecture, an introduction to various forecasting techniques and their main principles is given. The students now have to analyse the data coming from expert judgements, competitor's data and sales data from the past in order to set up a marketing plan including an analysis of the market, expected growth and forecast of demand using a number of forecasting techniques.

#### Day 3.

**Production modelling.** A lecture on production techniques and linear and integer programming is given prior to this day's assignment. If no linear/integer programming modelling background is available, this session can take from 3 up to 5 hours, and includes topics in the field of production and logistics (MRP, hiring/firing people, temporary work, inventory management). The assignment of the day is to build a production model to determine the monthly need for operators to fulfil the demand forecast specified on day 2.

#### Day 4.

**Cash flow analysis.** In a 1 hour session, the basic principles of cash flow analysis are reviewed and the need of integration of previous analyses is highlighted. In this assignment, students have to define a forecast of the need for cash for the next year, also including the presence of cash shortages that express the company's need for money in the short run. This assignment should be based on the analysis of the production plan (day 3) and the analysis of need for cash for raw materials, wages, temporary work, overtime work, etc.

#### Day 5.

**Executive report and presentation.** On this day, no lecture is given, but instead, the students have to present their final work to the bank. Supported by an executive summary of proposal to a bank (loan) and an oral and written presentation of their main results and strategy, they should aim at convincing the bank to invest the requested money in their company. The companies that receive the loan are invited for a drink, celebrating their victory and their hard work of the past 5 days.



# Course outline

## Decision Making for Business I

Improving your business processes to make timely and decisive actions

*“When you know better, you do better.” (Maya Angelou)*

### Decision Making training

The training “Decision Making for Business” is known in the academic world as Operations Research (OR). OR professionals use specialised tools and techniques to help organisations make better business decisions. It is a discipline that focuses on the **integration** of business processes with data analysis and automated decision support software systems. The trainings are based on **practical experience** as well as **academic research** published in international journals, written by Mario Vanhoucke.

#### Topic 1

**Theory of constraints.** Theory of constraints (TOC) is a management paradigm that views any manageable system as being limited in achieving more of its goals by a very small number of constraints. A production process case study will be used to introduce the students to business process modelling using linear programming techniques.

**Linear programming.** Linear programming (LP) is a mathematical method for determining a way to achieve the best outcome such as maximum profit or minimum cost in a given mathematical model for some list of constraints. An Excel based solution methodology will be discussed and shown by various business examples.

#### Topic 2

**Integer programming.** In order to make models more realistic and powerful, the integer programming (IP) technique will be discussed. Both exercises and practical applications based on real world experience will be shown, and a deeper look into the underlying optimisation mechanism will be part of this session.

#### Topic 3

**Case study I.** The students will be introduced to a case study focusing on integrating various business points of view to optimise decisions. After an introductory session to correctly interpret the available data and to highlight the importance to understand conflicting company points of view between e.g. accountants, marketeers and production managers, students will have to solve the business problem in groups.

#### Topic 4

**Consultancy experiences.** Depending on the background of the students (marketing, HRM, finance, airline scheduling, hospital scheduling, personnel optimisation, etc.), case studies and experiences on consultancy projects will be discussed in class, focusing on an integrated approach from data collection, model building, implementation and change management.

#### Topic 5

**Scenario analysis I.** This session extends the models of previous sessions to business environments with higher uncertainty and risk. Scenario analysis using the decision tree analysis (DTA) technique will be discussed and illustrated using a case study that has to be solved in groups.

**Scenario analysis II.** The results of the exercise made in groups will be discussed between the group members, followed by a general class feedback session.

#### Topic 6

**Simulation.** Since uncertainty typifies business, the decision making process must take certain unknown factors into account. Simulation can be a helpful tool to easily replicate a real world system on a computer in search of improvements and better decisions.

**Case study II.** A practical case study from the airline sector will be shown to illustrate both the ease and power of simulation.

### Software support

Students can have free online access to **MS Solver** ([www.solver.com](http://www.solver.com)) to download a free student version of Excel's optimisation engine in Windows and Mac. The optimisation software will be used throughout all class exercises and case studies and is easy to use in the students' favourite Excel environment.

### Who

The training is targeting young MBA professionals with or without a background and experience in modelling. All topics are relevant for managers working in both the private and public sector, and apply to supply chain managers, marketeers, project managers, financial engineers, human resource managers, and many others who are responsible for business processes with critical performance, time and budget targets.



# Course outline

## Decision Making for Business II

Improving your business processes to make timely and decisive actions

*“The common question that gets asked in business is, ‘why?’  
That’s a good question, but an equally valid question is, ‘why not?’” (Jeff Bezos)*

### Integrated Decision Making exercise

This data driven decision making course module was developed as an integrative case study containing six sequential cases A to F to manage and optimise the operations of a hospital. The case study is used in the Business Engineering programme at the faculty of Economics and Business Administration of Ghent University and requires advanced analytical skills and knowledge and understanding of Operations Research. It imitates the process of a data-driven consultant responsible for the optimisation of work at four departments of a hospital, and includes making decisions on the balance of the workload, the design of a new shift system, the allocation of budget among departments and the operational assignment of nurses to shifts. The students will work in small groups in order to thoroughly analyse a case study, and report their findings in a paper and professional presentation to a management of the hospital. Rather than presenting an optimal solution approach, they should be able to convince the management that their solution has potential to improve the current situation. The focus of the case study lies in the use of models in a real-life setting, showing the necessity of adapting and simplifying the models to make them easy to understand to a wide range of decision makers. Defining the problem, selecting the appropriate model and presenting the solution to a team are the key components of this case study. Moreover, teamwork and communicative skills should also belong to the core competences of each member of the team to bring this case study assignment to a good end (more info is available at [www.pmgamecenter.com](http://www.pmgamecenter.com)).

After this 10 to 12 weeks course module, the students should be able to

- Analyse a practical problem and translate it into an optimisation model, despite the overwhelming amount of non-structured data.
- Show the advantages and disadvantages of the different solution techniques, using non-technical terminology.
- Use optimisation software for the analyses of the problem under study, aiming at solving problems and presenting solutions to a team with different backgrounds.

### Case A.

**Demand modelling.** The first step in the case study consists of an extensive data analysis exercise, in which all tables of the case must be interpreted and the problem statement must be defined. At this stage, the student must determine how many staff is needed at different times over some planning/rostering horizon, looking at both the patient demand and nurse supply. Both sides need to be determined in terms of workload units in order to reveal the discrepancy between the two levels of workload. A linear programming model must be formulated to better match supply against demand, possibly leading to a decrease in the discrepancy.

### Case B.

**Budget partitioning.** Case B extends the analysis done for case A with a small number of calculation steps necessary to partition the overall budget over the different departments. To that purpose, the cost per workload unit needs to be estimated using a simple linear regression analysis and the budget needs to be re-allocated, if appropriate, by means of a small and straightforward linear programming model. An optional sensitivity analysis can be done to reveal the impact of changes in the data on the optimal allocated budget.

### Case C.

**Re-designing the nursing organisation.** In this part of the case, a sequence of models needs to be developed and compared in order to design a new shift system. Both the number of shifts, their length and their starting times must be questioned and optimised to better capture the imbalance between work availability and supply. The students must set up their own integer programming (IP) models and make a choice between running different models in order to obtain heuristic results or searching an optimal solution by setting up a complete integrated cost minimisation model. Afterwards, students have to interpret the solution and make possible adaptations to decide upon a final shift design system.

### Case D.

**Ward budgeting.** This exercise involves the creation of lines of work spanning the rostering horizon for each staff member. The process of constructing a line of work depends on the basic building blocks, i.e., the shifts chosen in the previous case C. This module involves the incorporation of both rules relating to lines of work and the pattern of demand. The former ensures the feasibility of individual lines of work while the latter ensures that all personnel members satisfy the work requirements at all times in the work horizon. A sequence of integer programming models have to be solved under different scenarios and in a dual phased approach. The solution of the first phase serves as input for the second phase in order to determine the number of nurses per skill type given the coverage requirements found in case C and the shift choices made there.

### Case E.

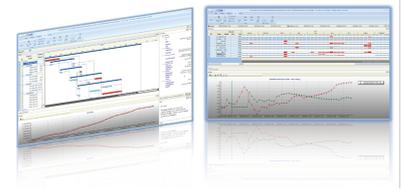
#### **Shift scheduling.**

This part of the case involves the assignment of individual staff to the lines of work or the immediate personalised construction of lines of work. Roster assignment involves the allocation of lines of work to individual staff members. During the assignment it is common to include individual staff preferences, availability and qualification as part of the process. A heuristic optimisation method needs to be developed in a programming language based on the heuristic solution approach proposed in the case. The computer program will need input calculated in the previous parts of the case and aims at the development of an operational nurse roster that satisfies various objective and subjective targets. The roster is the final output of the case and can be evaluated based on different criteria.

### Case F.

**Determining the size of the float unit.** A final step in the case study consists in a what-if analysis that puts the operational schedule of case E in the right perspective. While the schedule produced in case E relies on the optimal input values found in the previous cases, it might and will be subject to unexpected changes due to various reasons, such as employee sickness, changing demand, etc. It lies in the task of the students to be prepared for these unexpected events, and to analyse the potential impact of these changes on their outcome and objectives.





# Course outline

## Project Management

Dynamic scheduling: Baseline scheduling, risk analysis and project control

*“When you know better, you do better.” (Maya Angelou)*

### Project Management training

Dynamic scheduling is a Project Management (PM) discipline that focuses on the **integration** of baseline scheduling, risk analysis and project control. The trainings are based on **practical experience** as well as **academic research** published in “Measuring Time” and “Dynamic Scheduling”, written by Mario Vanhoucke and published by Springer ([www.or-as.be/bookstore](http://www.or-as.be/bookstore)).

### Introduction

**Introduction.** A general introduction to Project Management using Dynamic Scheduling is necessary to understand all upcoming topics of the course. Concepts such as project life cycle management, project mapping and knowledge about the project objectives are key to understand all future sessions.

### Topic 1

**Network analysis.** The construction of a project network consisting of activities and relations between activities and the use of scheduling techniques such as the PERT/CPM method and its advanced extensions are basic needs to any project manager. Concepts such as the critical path, activity slack, activity crashing, and other scheduling techniques are discussed.

### Topic 2

**Business game.** The techniques and concepts discussed in the morning session are now used in a computerised exercise published in the Project Management Journal. The computer exercise consists of an introduction, a 90 to 120-minutes game session where the participants have to analyse the current project performance, analyse alternatives and finally take decisions at each decision moment, and a feedback session.

### Topic 3

**Risk analysis.** A baseline schedule constructed by the techniques discussed on day 1 is only relevant when the impact of uncertain events is measured and analysed. Schedule risk analysis is such a technique that reviews the constructed schedule in the light of uncertainty. An overview of the risk analysis techniques and their use in practice is given.

### Topic 4

**Resource allocation.** Allocating resources to projects increases the complexity of constructing a resource-feasible baseline schedule. Knowing the scheduling objectives, understanding the relation between the precedence and resource constraints as well as being aware of the software strengths and weaknesses are key to find a realistic and workable schedule. Various techniques and tools will be discussed in class and illustrated by real-life project examples.

### Topic 5

**Project control.** Controlling projects is key to the success of a project since it enables the project manager to measure the performance of the project in progress and gives early warnings as triggers for corrective actions. An introduction to Earned Value Management and Earned Schedule control techniques is given, and a critical overview on the use of standard and newly developed techniques is discussed.

### Extra topics

**Advanced topics, state-of-the-art research and exercises.** The course will be supported by guest lectures on advanced PM topics, state-of-the-art research methods and exercises on the techniques discussed in previous sessions. The specific topics differ each year and depend on the currently best known methods and best practices.

**Note.** An integrated **software exercise** in the commercial software tool ProTrack (demo version) is given to put the learning objectives of the PM training into practice. The training will be evaluated based on a written closed-book exam as well as a group work consisting of a presentation and a written report.

## Online support

Students can have free online access to **PM Knowledge Center** ([www.pmknowledgecenter.com](http://www.pmknowledgecenter.com)) that brings relevant articles and references on dynamic scheduling together at one place! PMKC is made by OR-AS and supported by software tools such as ProTrack ([www.protrack.be](http://www.protrack.be)), the Project Scheduling Game ([www.or-as.be/psg](http://www.or-as.be/psg)) and P2 Engine ([www.p2engine.com](http://www.p2engine.com)) that can be used during the training sessions. A free app **ORASTalks** is also available on iOS, Android and Windows Phone to facilitate and stimulate interaction and to share content with the students ([www.or-as.be/orastalks](http://www.or-as.be/orastalks)).

## Who

The training is targeting young **PM students** and **professionals** with or without a background and experience in Project Management. All topics are relevant for project managers working in both the private and public sector, and apply to large and small projects with critical performance, time and budget targets.

## Study material

The **student handbook** is "Project Management with Dynamic Scheduling: Baseline Scheduling, Risk Analysis and Project Control", written by Mario Vanhoucke and published by Springer.

# Nice to know

# Solving problems using software

## Good old MS Excel and its Solver



*“Never bring the problem solving stage into the decision making stage. Otherwise, you surrender yourself to the problem rather than the solution.” (Robert Schuller)*

### MS Solver

Good old Excel. While a lot of optimisation tools are available on the market, MS Excel is still the one that is easy to use and accessible to anyone. And it has a solver for LP and IP problems, called MS Solver. In Excel 2010, a number of improvements to MS Solver have been made that make it easier for beginners, including a renewed graphical user interface, that will be briefly shown and discussed below and illustrated in the picture. Users of the older MS Solver version should take a look at the various internet sources. However, the basic idea and solution approach remains unchanged.

The screenshot shows the 'Solver Parameters' dialog box in MS Excel. It includes fields for 'Set Objective' (containing '\$F\$12'), 'To:' (with radio buttons for 'Max', 'Min', and 'Value Of:'), and 'By Changing Variable Cells'. Below these are 'Subject to the Constraints' with 'Add', 'Change', 'Delete', 'Reset All', and 'Load/Save' buttons. A checkbox for 'Make Unconstrained Variables Non-Negative' is checked. A 'Select a Solving Method' dropdown is set to 'GRG Nonlinear', with an 'Options...' button next to it. A 'Solving Method' text box provides instructions on when to use the GRG Nonlinear, LP Simplex, or Evolutionary engines. At the bottom are 'Close' and 'Solve' buttons. Arrows point from text labels on the right to these specific elements in the dialog box.

- Objective function**  
(single cell, behind this cell there is a (linear) equation)
- Decision variables**  
(empty cells in the MS Excel sheet)
- Constraints**  
(Add here the (linear) equations)
- Non-negativity constraints**  
(Add them by a single click!)
- Take a look at the options!**  
(Some of them are important)
- Solution approach**  
(Use SIMPLEX to solve LP and IP models)

The choice of the solution approach is crucial when solving a problem, since you need to tell to MS Solver whether you want to solve LP or IP problems (to return the optimal solution) or rather nonlinear problems (without the guarantee to have the best solution). The three choices are:

- **Simplex Method:** The Simplex method is used for solving linear problems, and always returns the best possible solution (i.e. the optimal solution).
- **GRG Nonlinear:** The GRG solver is used for solving smooth nonlinear problems. Smooth nonlinear equations are equations such as  $\max(x_1, x_2)$  or  $\log(x_1)$ . These equations are certainly nonlinear, but they do not contain a “jump” as is the case for e.g. absolute values and others. Beware, the solution reported by GRG Solver will probably not be the best possible solution (i.e. heuristic, not optimal).
- **Evolutionary Solver:** The new evolutionary solver accepts Solver models defined in exactly the same way as the Simplex and GRG Solvers, but uses genetic algorithms to find its solutions (cf. “What is nonlinear programming?” in this book). This Evolutionary Solver can be used for any MS Excel formula or function, beyond the smooth nonlinear equations. Spreadsheet functions such as IF and ABS fall into this category. Beware, the solution reported by the evolutionary Solver will probably not the best possible solution (i.e. heuristic, not optimal).

# Nonlinear transformations

## How to model absolute values?



*"I don't believe that life is linear. I think of it as circles - concentric circles that connect."*

*(Michelle Williams)*

### Absolute value in linear programming

Many of my students at Ghent University (cf. "Decision Making for Business II" in this book) or Vlerick Business School (cf. "Decision Making for Business I" in this book) struggle with using absolute values in linear programming.

Since the use of absolute values is not linear, many of the students tend to use the big M method (cf. "How to use the big M method?" in this book), but that is - although possible - not necessary. It can easily be done by defining two new so-called slack variables (I will call them  $s^+$  and  $s^-$ ) for each absolute value equation.

Let me give you an example with only one absolute value equation:

Suppose that the absolute value of two variables,  $x$  and  $y$ , must be taken, as follows:  $|x - y|$ , then the LP program can be formulated as follows:

$$\begin{aligned} & \min s^+ + s^- \\ & \text{subject to} \\ & y - x \leq s^+ \\ & x - y \leq s^- \end{aligned}$$

Just check whether this is correct by using  $x = 5$  and  $y = 10$  (absolute value = 5) or  $x = 10$  and  $y = 5$  (absolute value also = 5). The model will always give you 5 (i.e. the absolute value) in the objective, as shown below:

$$\begin{aligned} x = 5 \text{ and } y = 10 \text{ then } s^+ = 5 \text{ and } s^- = 0 \\ x = 10 \text{ and } y = 5 \text{ then } s^+ = 0 \text{ and } s^- = 5 \end{aligned}$$

Since the objective minimises the sum of the slack variables  $s^+$  and  $s^-$ , only one of them will be positive, and the other will always be equal to zero. The variable with the positive value will return the absolute value.

How elegant, isn't it?



# Nonlinear transformations

## How to use the big M method?

*"I have a different way of thinking. I think synergistically. I'm not linear in thinking, I'm not very logical." (Imelda Marcos)*

### Big M method in linear programming

In my courses at Ghent University (cf. "Decision Making for Business II" in this book) or Vlerick Business School (cf. "Decision Making for Business I" in this book), I explain my students how the big M method works, and I ask them to model the equation  $y \leq \max\{x_1, x_2\}$  in a linear programming model.

$$\begin{aligned} &\text{How to maximise } y \\ &\text{subject to} \\ &y \leq \max\{x_1, x_2\} \end{aligned}$$

The M represents some very large number, larger than any possible value of the decision variables. Don't use an M in your MS Excel file. Use a number. A large number.

### Solution 1

The problem can be linearised by using 4 equations, and by defining a new decision variable, say w, which is assumed to be a binary (only values 0 or 1) variable. Then the four equations look like these:

$$\begin{aligned} x_1 &\leq x_2 + M * w \\ x_2 &\leq x_1 + M * (1 - w) \\ y &\leq x_1 + M * (1 - w) \\ y &\leq x_2 + M * w \end{aligned}$$

You can easily check why this set of constraints replaces the nonlinear equation. Just set  $w = 0$  and  $w = 1$  and look each time how the constraints look like. You will see they all satisfy the condition  $y \leq \max\{x_1, x_2\}$ , as you can see below:

<u>w = 0</u>	<u>w = 1</u>
$x_1 \leq x_2$	$x_1 \leq x_2 + M$
$x_2 \leq x_1 + M$	$x_2 \leq x_1$
$y \leq x_1 + M$	$y \leq x_1$
$y \leq x_2$	$y \leq x_2 + M$

In case  $w = 0$ , only constraints 1 and 4 are relevant, since the two other constraints are always non-binding (due to the big M). These two relevant constraints state that y is smaller than  $x_2$ , which is thanks to constraint 1, the maximum of  $x_1$  and  $x_2$ . Likewise, in case  $w = 1$ , the first and last constraints are now non-binding and irrelevant thanks to the big M value, but y is now smaller than  $x_1$ , which is the maximum of  $x_1$  and  $x_2$  as modelled by constraint 2.

Some of my students ask me: How do you come up with such a solution? Well, to be honest, I don't know. I guess it's not that difficult, but it's more trial and error than using a predefined approach. And every time you try, you might come up with an alternative, but equally good solution. Look at solution 2!

### Solution 2<sup>(a)</sup>

As usual, many alternative approaches exist. Everybody should know that  $y \leq \max\{x_1, x_2\}$  is not a linear equation but it is the same as  $y \leq \min\{x_1, x_2\} + |x_1 - x_2|$ . Note that  $|x_1 - x_2|$  is used to refer to the absolute value of  $x_1 - x_2$ .

The first part is easy to linearise, since  $\min\{x_1, x_2\}$  can be replaced by two constraints,  $y \leq x_1$  and  $y \leq x_2$ , and so the equation will become like this:

$$y \leq x_1 + |x_1 - x_2|$$

$$y \leq x_2 + |x_1 - x_2|$$

But the problem is that the absolute value  $|x_1 - x_2|$  is not linear as well.

However, making absolute values linear is simple, just take a look at “Nonlinear transformations: How to model absolute values?” in this book.

**Conclusion:** You can easily make this nonlinear absolute value linear, and by integrating everything, you should be able to build a model that exactly does what it needs to do.

How elegant, isn't it?

<sup>(a)</sup>Author of this solution: One of my students in the FT BiMBA at Peking University (2013)

Ce (Jack) Wang

MBA Candidate

Beijing International MBA at Peking University (BiMBA)

National School of Development (NSD)

Peking University

Thank you Jack!

# Non-negativity constraints

Shadow prices = reduced costs?



*“Intuition is a suspension of logic due to impatience.”*

*(Rita Mae Brown)*

## Why is the shadow price of a non-negativity constraint equal to the variable’s reduced cost?

In the chapter “Deterministic models: What is sensitivity analysis?” of this book, I explained what the shadow price and reduced cost of a linear programming model really mean. I wrote the following:

- A shadow price value is associated with each constraint of the model. It is the instantaneous change in the objective value of the optimal solution obtained by changing the right hand side constraint by one unit.
- A reduced cost value is associated with each variable of the model. It is the amount by which an objective function parameter would have to improve before it would be possible for a corresponding variable to assume a positive value in the optimal solution.

Then, I continued at the end of this chapter with a sentence as follows:

**Statement:** “It should be intuitively clear that the reduced cost is equal to the shadow price of the non-negativity constraint of the variable”.

Intuitively clear? Really? I suggested the reader to think about this statement, but some people asked me to explain this statement in more detail. I guess it is less intuitive than I initially thought. Therefore, below you find some explanation. I still think, however, that it is best not to read this and think about it for a minute. It’s not that difficult after all.

Let’s take a look at the model (I call it here model 1) that I use in the sensitivity chapter. I assume that the decision variables  $x_1$  and  $x_2$  are amounts for production of ... whatever.

### Model 1.

minimise cost  $[C] = 10 x_1 + 7 x_2$

subject to the following constraints:

$$x_1 + x_2 \geq 10$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$

The optimal solution is equal to  $x_1 = 0$  and  $x_2 = 10$  with an objective of  $C = 70$ .

In the sensitivity chapter I explain that the reduced cost for  $x_1$  is equal to 3. Indeed,  $x_1$  is too expensive compared to  $x_2$ , and therefore  $x_1 = 0$ . Therefore, the cost should be reduced from 10 to 7 (or lower, so by minimum a value of 3) to make the production of  $x_1$  attractive, hence, the value of 3 for the reduced cost.

Now let’s go back to the statement:

**Statement** (illustrated on model 1): “The reduced cost of a decision variable (i.e. value 3 for variable  $x_1$ ) is equal to the shadow price of the non-negativity constraint of the variable (i.e.  $x_1 \geq 0$ )”

The shadow price for the constraint  $x_1 \geq 0$  can be defined as follows: If you increase the right hand side of that constraint (currently 0) by one unit (i.e. the constraint changes to  $x_1 \geq 1$ ), what is the impact on the objective. Hence, the model changes into (notice the small difference):

**Model 2.**

minimise cost  $[C] = 10 x_1 + 7 x_2$

subject to the following constraints:

$$x_1 + x_2 \geq 10$$

$$x_1 \geq 1$$

$$x_2 \geq 0$$

The optimal solution of model 2 is now equal to  $x_1 = 1$  and  $x_2 = 9$  with an objective of  $C = 73$ . This is exactly 3 units (e.g. Euro) more than the previous solution, and hence, the shadow price of the constraint  $x_1 \geq 0$  in model 1 is equal to 3. This value 3 is equal to the reduced cost of  $x_1$  in model 1, which illustrates my statement.

How elegant, isn't it?

# The more you practice, the more you can

## Linear and integer programming examples



*“For the things we have to learn before we can do them, we learn by doing them.” (Aristotle)*

As for anything in life, practicing is the only way to gain expertise (I can assure you that my accordion lessons require a lot of practicing. Hours and hours the same set of notes, until suddenly, I play it and wonder why on earth it was so damn difficult the day before). And so it is for formulating LP and IP models. The beauty of modelling lies in the fact that it is each time different. Of course, you can rely on a number of standard tricks, but every time, the solution to a problem requires a deep understanding of the characteristics of the problem. It's a fascinating world, the world of modelling.

In order to practice, one needs exercises. Below, you can find eight extra examples of LP and IP, including solutions, taken from various sources in literature.

Linear programming models:

- Blending problem: Use four raw grains and blend them to produce dry pet food.
- Float-away tours problem: Purchase new rental boats for hiring during the summer within a limited budget.
- Portfolio planning problem: Invest your money over the next four months to maximise the total interest earned.
- Recycling solid waste problem: Collect four types of solid waste material, treat them and then recycle them into a saleable product.

Integer programming models:

- Capital budgeting problem: Invest your money in six possible projects.
- Cutting stock problem: Optimise the production of tin cans which are stamped from metal sheets.
- Less-than-truckload shipping problem: Optimise the delivery of loads on a daily basis to five customers for a trucking company.
- Product line investment: Expand the operations of a company by evaluating seven new potential product lines.



Download the examples (exercise and solutions) from [www.or-as.be/blog/operations\\_research\\_examples](http://www.or-as.be/blog/operations_research_examples)

# The millennium prize problems

An 'easy' way to get rich!



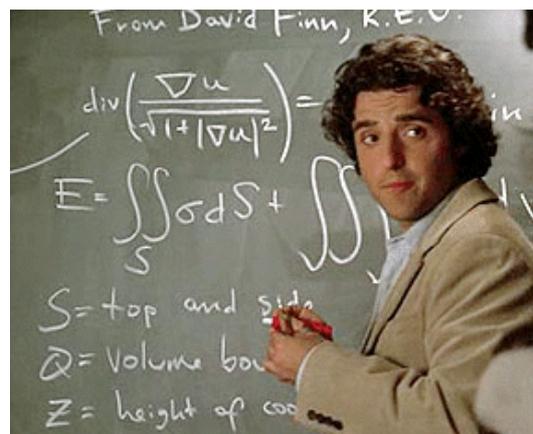
*"If you don't work on important problems, it's not likely that you'll do important work."* (Richard Hamming)

The **Millennium Prize Problems** are 7 problems in mathematics that were stated by the Clay Mathematics Institute in 2000. Today, 6 of the 7 problems remain unsolved. A correct solution to any of the problems results in a \$1,000,000 prize (sometimes called a Millennium Prize) being awarded by the institute. An easy way to get rich! Well, easy...

The word NP-hard has already been used in this book to refer to the complexity of solving particular problems. The P versus NP concept is not easy, and details are certainly outside the scope of this book. Basically, the set of P problems refers to a set of problems for which some algorithm can provide an answer in polynomial time (hence P). In a very non-scientific way, I would call polynomial time just "fast". Most familiar mathematical operations such as addition, subtraction, multiplication, and division, as well as computing square roots, powers, and logarithms, can be performed in polynomial time. Computing the digits of most interesting mathematical constants, including pi, can also be done in polynomial time. However, for some questions, there is no known way to find an answer quickly, but if one is provided with information showing what the answer is, it is possible to verify the answer quickly. The class of questions for which an answer can be verified in polynomial time is called NP. Most mathematicians believe P is not equal to NP, but until today, no one has a proof for it.

The implication of  $P = NP$  would mean that problems that were previously thought to be extremely hard to solve can now be solved in polynomial time, and everything would be much more efficient. Transportation would be scheduled optimally to move people and goods around quicker and cheaper. Manufacturers would be able to improve their production to increase speed and create less waste. And much much more...

Scott Aaronson from MIT said that "if  $P = NP$ , then the world would be a profoundly different place than we usually assume it to be. There would be no special value in 'creative leaps', no fundamental gap between solving a problem and recognising the solution once it's found. Everyone who could appreciate a symphony would be Mozart, everyone who could follow a step-by-step argument would be Gauss". I guess you get the idea. This is no simple stuff. You can try and start doing the maths to find the proof and walk out your house with a million dollar in your hands, but I guess you better keep doing what you do.



Dr. Charlie Eppes (real name David Krumholtz) in NUMB3RS

One of the most famous researchers working on the problem (although not in reality) is Charlie Eppes who helps his brother Don solve crimes for the FBI in the American crime drama television series NUMB3RS. In-between solving crimes, he works on all kinds of mathematical problems, including the P and NP problem, without much success.

# Library

# Nassim Nicholas Taleb

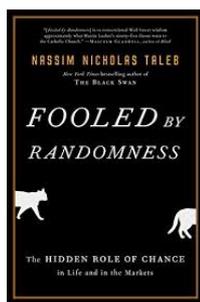
Embrace randomness and don't be fragile

*"Option sellers eat like chickens and go to the bathroom like elephants."*  
(Nassim Nicholas Taleb)

## Thinking about uncertainty

Unlike what most people believe, I am convinced that Taleb is a true optimist, with a positive mindset and view on life. His books might show the contrary, but I don't read them like this. Taleb tells how randomness is a big part of life, and how people underestimate the role of randomness. If you ever think you've made it in business because you could predict the future much better than others, you better realise you were just very lucky! His books received a mixture of praise and criticism. Taleb's writing style and his representation of the statistical literature is something you like or hate. He probably is not the most friendly guy in the room, but has a clear and outspoken opinion. And I like his style!

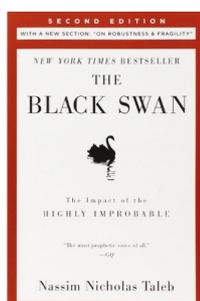
Taleb's four volume philosophical essay on uncertainty, titled the Incerto, covers the following books: Fooled by Randomness (2001), The Black Swan (2007–2010), The Bed of Procrustes (2010), and Antifragile (2012). For me, it was the start of a wonderful journey into the role of randomness in decision making. And as always, the journey was the reward!



### **Fooled by Randomness**

The Hidden Role of Chance in Life and in the Markets

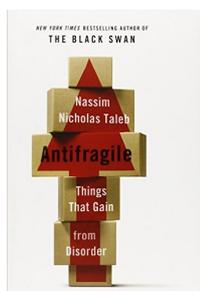
This book is my all time favourite. It changed my view on many things in life, and it certainly changed my way of teaching "decision sciences" course modules. What an inspiration! In his book, Taleb discusses how humans tend to explain random outcomes as non-random. They overestimate causality, and view the world as more explainable than it really is. He explains that we shouldn't spend too much attention to the winners (Steve Jobs, Mark Zuckerberg, ...) and try to learn from them, while forgetting the huge number of losers. It's often just luck!



### **The Black Swan**

The Impact of the Highly Improbable

In his excellent book, Taleb refers to the black swan theory as a metaphor that describes an event that comes as a surprise, has a major effect, and is often inappropriately rationalized after the fact with the benefit of hindsight. He describes that these rare and improbable events do occur much more than we dare to think. His book is an overview of illustration of our blindness with respect to randomness, particularly large deviations. He tells the nice story about a turkey, but discusses interesting topics such as the narrative fallacy, the irrelevance of history to predict the future, and much more. This is a must-read for... well... everyone!



### **Antifragile**

Things That Gain from Disorder

Taleb opens his book with the following words "Some things benefit from shocks; they thrive and grow when exposed to volatility, randomness, disorder, and stressors and love adventure, risk, and uncertainty. Yet, in spite of the ubiquity of the phenomenon, there is no word for the exact opposite of fragile. Let us call it antifragile. Antifragility is beyond resilience or robustness. The resilient resists shocks and stays the same; the antifragile gets better." Do I need to write more? It's a super-mega-extra-ordinary masterpiece, and I certainly will read it again and again and again.

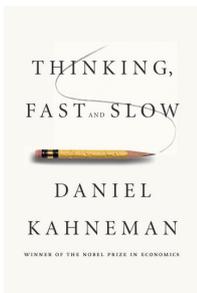
# Daniel Kahneman

## The riddle of experience versus memory

*“If individuals are rational, there is no need to protect them against their own choices.”*  
(Daniel Kahneman)

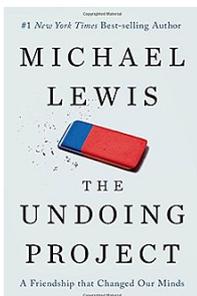
### The two selves

As a winner of the Nobel Memorial Prize in Economic Sciences (2002), the Israeli-American psychologist Daniel Kahneman is known and highly regarded for his excellent work on the psychology of judgment and decision-making. While I initially thought that psychology is not really my cup of tea, I might have to review that now. In the first part of his book, I thought I would give up, since there seemed to be hardly any link with my “decision making for business” work. He distinguishes between the remembering and experiencing self, which initially were very abstract concepts to me. However, the second part of his book was not only an eye opener, but a door to a new world! Inspiring and relevant from the first to the last word. I use it in all my quantitative teaching sessions ever since.



#### Thinking, Fast and Slow

This is an excellent book about decision making. Not from a modeling point of view, but from a human being point of view. It's a book full of intellectual surprises. Kahneman tells about heuristics and biases and offers explanations for why humans struggle to think statistically. He introduces concepts such as the anchoring effect, framing effect, sunk cost fallacy, and even elaborates on rationality and happiness. Concepts that are not only interesting to read, but highly relevant in a “decision making for business” context! He discusses the tendency for problems to be addressed in isolation and how, when other reference points are considered, the choice of that reference point has a disproportionate impact on the outcome.



#### The Undoing Project

The Undoing Project explores the close partnership of Israeli psychologists Daniel Kahneman and Amos Tversky, whose work on heuristics in judgment and decision-making demonstrated common errors of the human psyche, and how that partnership eventually broke apart. The book discusses the same topics as in “Thing, fast and slow”, but now investigates them under the umbrella of friendship and partnership, but also jealousy and competition. Highly recommended!

# Alan Turing

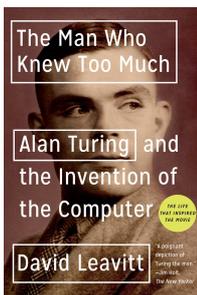
Meet the father of the digital world

*"Turing believes machines think,  
Turing lies with men  
Therefore machines do not think."  
(Alan Turing)*

## The father of the digital world

If you like Operations Research, you have an interest in Computer Science and/or you consider yourself as a kind of computer nerd, you know Alan Turing. To all the others: it's a shame you don't know him, since he's the one who invented the iPhones, iPad, computers and much more stuff for you!

Alan Turing was a brilliant mathematician. Born in London in 1912, he studied at both Cambridge and Princeton universities. In 1936, Turing published his paper "On Computable Numbers, with an Application to the Entscheidungsproblem" and that paper changed the world for good! Turing proved that a universal computing machine, now known as the Turing machine, would be capable of performing any conceivable mathematical computation if it were representable as an algorithm. The inventor of the algorithm! The father of the computer! He is the founder of big data analysis! He has cracked the Enigma code. The Enigma was a type of enciphering machine used by the German armed forces to send messages securely during the world war. Thanks God Turing cracked the code!



### The Man Who Knew Too Much

Alan Turing and the Invention of the Computer

This book described the life of the key founder of theoretical computer science and artificial intelligence. It tells the story of how this genius wrote his first paper without realizing that he was changing the world. It also tells the story of how his homosexuality was at that time a criminal act. It tells about how the world war 2 was won thanks to his never-ending search for algorithms. It's a story of ingenuity, computer models in a world without computers, sadness and suicide. It's a story about a man who, just like Albert Einstein, Isaac Newton, or any other well-known genius, deserves our endless respect!



### The Imitation Game

The Imitation Game, a movie about the life of Alan Turing (Benedict Cumberbatch), was a commercial and critical success and shows Turing's character and relationships and brings his legacy to a wider audience. Turing committed suicide in 1954 at the age of 41, due to the scandalous treatment of the mathematical genius by the British government. His homosexuality was at that time a forbidden crime, and what happened to Turing was utterly barbaric. The genius mathematician should be celebrated for his genuity and his achievements. We should all be grateful to Turing, and we should never forget his legal victimisation. He deserved so much better.

### NP Hard

It is difficult to solve



A problem can be "easy" to solve [P] or very hard [NP]. The American-Canadian computer scientist and mathematician Stephen Cook has made major contributions to the fields of complexity theory and proof complexity. He is the winner of the Turing award. The  $P = NP$  problem is still an unsolved problem, and belongs to the class of the Millennium Prize Problems (meaning: Solve it, and you never have to work again!) During my stay in November 2015 at Peking University, I was teaching a course module on "Decision Making for Business" and I occasionally explained my fantastic students that many problems are so-called NP hard (i.e. not easy to solve). Everytime again, I mentioned the word in-between my business examples, and at the end of my course module, they surprised my with a cake with the words "NP Hard Solver", as you can see here next to the text! How cute! Much appreciated!

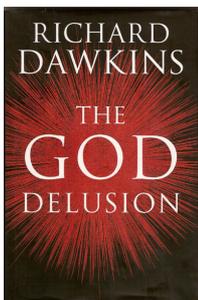
# Richard Dawkins

The existence of the statistically improbable

*“Complex, statistically improbable things are by their nature more difficult to explain than simple, statistically probable things.”*  
(Richard Dawkins)

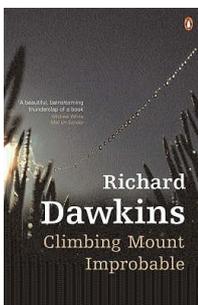
## An ode to Darwin

Richard Dawkins probably is the biggest fan of Charles Darwin and his theory of evolution. In all his books, he expresses his admiration for the natural evolution (Darwinism), and sums up numerous arguments why there is almost certainly no God. In his typical lyrical and metaphorical style, he not only describes the beauty of nature and everything around us, but he also makes the connection to decision making techniques (more precisely, the non-linear optimisation techniques known as metaheuristics). If you ever wonder what the origin of species is, read his books and be impressed and speechless! If you want to optimise your business processes (which is hopefully why you have downloaded this “Taking Sound Business Decisions” book), learn how nature operates!



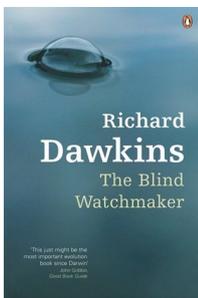
### The God Delusion

In this book, Richard Dawkins excellently describes why a supernatural creator almost certainly does not exist, and he therefore qualifies the belief in a personal god as a delusion. He argues that there exists a strong body of evidence of the non-existence of such a God, and he illustrates his statement with numerous examples. With quotes such as “when one person suffers from a delusion it is called insanity. When many people suffer from a delusion it is called religion”, he explains that moral behaviour does not require the existence of a god, and therefore, the roots of morality should and can be explained in non-religious terms.



### Climbing Mount Improbable

This book tells the story about the theory of evolution, and about the probability of naturalistic mechanisms like natural selection. The metaphorical treatment is of a mountain, upon which evolution can only ascend in a gradual way, not being able to climb cliffs. In the book, Dawkins gives ideas about a seemingly complex mechanism coming about from many gradual steps that were previously unseen. It's simply one of the most wonderful books I have ever read. He tells how the shape of shells can be defined, and he devotes the last chapter to an explanation of how figs and wasps must have evolved together because neither figs nor wasps can reproduce without each other. If you want to be surprised by the beauty and ingenuity of nature, go to the bookstore and get this book!



### The Blind Watchmaker

Why the Evidence of Evolution Reveals a Universe Without Design

Much similar to his other books, Dawkins tries to convince the reader that what we see around us, is the consequence of the operation of natural selection upon random mutations over an immense period of time. He even describes a computer experiment with a simple evolutionary modelling program that, in 29 generations, started to provide two-dimensional biomorphs that looked like bats, spiders, scorpions, tree frogs and even a fox. Many algorithms used in “decision making for business” make use of similar principles (genetic algorithm, ant colony optimisation, ...) What a wonderful book!

“I could not imagine being an atheist at any time before 1859, when Darwin's Origin of Species was published.”

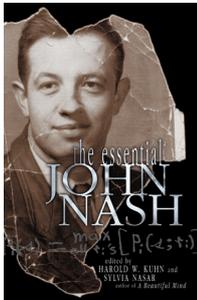
# John Nash

Life is a game

*“As you will find in multivariable calculus, there is often a number of solutions for any given problem.”*  
(John Nash)

## A genuine dilemma

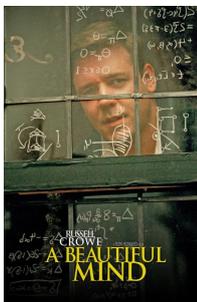
John Nash was an American mathematician and a Nobel Laureate in Economics who made fundamental contributions to game theory and has provided insight into the factors that govern chance and decision making inside complex systems found in daily life. His game theory concepts are crucial in understanding decision making processes, and as is often the case for a true genius, he suffered from mental illness after his main discoveries which made his life difficult and sad. I had the opportunity to meet him at the EURO Conference in July 2010 in Lisbon (Portugal) where he gave an overview of his work. I must say I was overwhelmed and impressed to meet the founder of decision making. Sadly, 5 years later, Nash and his wife were killed in a car accident in New Jersey on their way home from the airport after a visit to Norway.



### The Essential John Nash

Edited by Harold W. Kuhn & Sylvia Nasar

The book gives an overview of the man behind the concepts "Nash equilibrium" and "Nash bargaining" that have major relevance in not only economics but in any decision making context. John Nash has lived in the shadow of a condition diagnosed as paranoid schizophrenia, and has gone from being a wunderkind at Princeton and a rising mathematical star at MIT to the depths of mental illness. The book beautifully expresses the genius of Nash written by his friends and colleagues. It's an ode to a beautiful mind!



### A Beautiful Mind

The story begins in the early days of John Nash (Russel Crow) as a graduate student at Princeton University. Nash begins to develop paranoid schizophrenia and endures delusional episodes while painfully watching the loss and burden his condition brings on his wife and friends. The movie was shot when he was still alive and ends in a postive way. It's a beautiful piece of art, in which Nash is portrayed as a mathematical genius in search for a breakthrough. It's shows his obsession for mathematics, his paranoid schizophrenia, the side effects of medication and his endless for his wife.

# Eliyahu Goldratt

## Theory of constraints

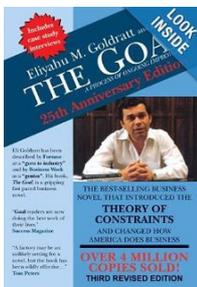
*“An expert is not someone who gives you the answer, it is someone who asks you the right question.”*  
(Eliyahu Goldratt)

### A chain is as strong as its weakest link

Eliyahu Goldratt was an Israeli business management guru, and the founder of the Theory of Constraints. This theory can be applied to many business processes, ranging from manufacturing to project management, and identifies the bottlenecks in a system as that part that determines how your targets could and should be reached. It's a highly relevant conceptual theory that I use for explaining the “Decision Making for Business” tools discussed earlier in this book, such as linear programming, integer programming, and much more! He's the author of several non-scientific books. Indeed, he wrote novels. Nice stories, with some underlying business concepts.

#### The Goal

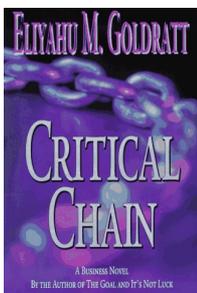
A Process of Ongoing Improvement



The book discusses how bottlenecks (constraints) in a manufacturing process should be identified, and how it is possible to reduce the impact of these constraints on the company objectives. It provides the reader with a useful tool for measuring and controlling the flow of materials.

The book is written as a novel. The main character is Alex Rogo, who manages a production plant owned by UniCo Manufacturing, where everything is always behind schedule and things are looking dire. During the interesting and joyfull conversations, Alex starts a wonderful search to turn the company's operations around from being unprofitable and unreliable to being successful. Every MBA student should read this book!

#### Critical Chain



This book is a story about a professor trying to attain his tenure at a university's business school. But as is always the case with the novels of Goldratt, the lesson to learn behind this story is a lesson in Project Management! My main area of research

Critical chain project management is a novel method of planning projects that emphasizes the presence of scarce resources required to execute project activities. It differs from more traditional methods, such as the critical path method, which mainly focus on network analysis and rigid scheduling. A critical chain project network strives to keep resources levelled, and requires that they be flexible in start times. Interesting eye-opener, with some (over-)simplifications that deserve some attention.

# Mario Vanhoucke

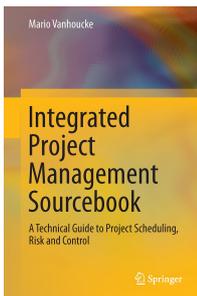
Being a project manager is the most beautiful job in the world

*"If there's a book that you want to read, but it hasn't been written yet, then you must write it."*

*(Toni Morrison)*

## I accept, with humility

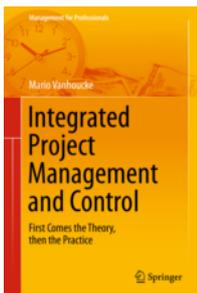
I love the process of writing! I accept that the previous discussed people live in a world where I don't belong. I accept that I do not have the intelligence that they have. But it nevertheless never kept me away of writing my own books.



### **Integrated Project Management Sourcebook**

A Technical Guide to Project Scheduling, Risk and Control

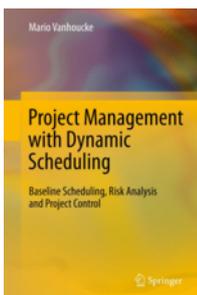
This book is intended to be the Integrated Project Management Sourcebook for students of any Project Management course focusing on the integration between baseline scheduling, schedule risk analysis and project control. The book is intended to be a technical guide rather than a management book that contains a set of +70 articles classified in the three main book themes, and each article contains links to other relevant articles of the book. The articles are accompanied with a set of questions to enable the reader to test his/her knowledge and understanding of the article's theme, for which the answers are provided at the end of this book. [Published in 2016].



### **Integrated Project Management and Control**

First Comes the Theory, then the Practice

This book presents an integrated approach to monitoring projects in progress using Earned Value/ Earned Schedule Management and Schedule Risk Analysis. Monitoring projects involves processes for identifying potential problems in a timely manner. When necessary, corrective actions can be taken to exploit project opportunities or to get failing projects back on track. Monitoring the performance of projects in progress requires a set of tools and techniques that should ideally be combined into a single integrated system. The book offers a valuable resource for anyone who wants to understand the theory first and then to use it in practice with software tools. [Published in 2014].



### **Project Management with Dynamic Scheduling**

Baseline Scheduling, Risk Analysis and Project Control

The topic of this book is known as dynamic scheduling, and is used to refer to the construction of a baseline schedule and the analysis of a project schedule's risk as preparation of the project control phase during project progress. It took me more than 10 years to write this book! But I believe it was worth the effort. The construction of a project baseline schedule is a central theme throughout the various chapters of the book. The creation of an awareness of the weak parts in a baseline schedule is also discussed. The baseline schedule and the risk analyses can be used as guidelines during the project control step where actual deviations can be corrected within the margins of the project's time and cost reserves. [Published in 2012. Second edition available since 2014].



### **Measuring Time**

Improving Project Performance using Earned Value Management

This book has resulted in my 15 minutes of fame in Rome in Italy during my presentation to a huge audience of thousands of project managers, and has defined the rest of my career. The book is the result of a project control research study awarded by the Belgian chapter of the Project Management Institute and the International Project Management Association. The book is meant to complement rather than compete with the existing books on the subject, and it deals with the project performance and control phases of the project life cycle to present a detailed investigation of the project's time performance measurement methods and risk analysis techniques in order to improve the corrective actions decision-making process during project tracking. [Published in 2010].

# References

# References

## Distributing academic research and scholarship

*"If you have a garden and a library, you have everything you need." (Marcus Tullius Cicero)*

Below you find a list of mainly peer reviewed papers, but also non-peer reviewed articles and book chapters as well as books in the field of project management and scheduling with a clear focus on using mathematical models for constructing project schedules, analysing risk and controlling projects.

### Peer reviewed international articles

Kerkhove, L.-P., Vanhoucke, M. and Maenhout, B., 2017, "On the resource renting problem with overtime", *Computers and Industrial Engineering*, 111, 303–319.

Martens, A., and Vanhoucke, M., 2017, "The integration of constrained resources into top-down project control", *Computers and Industrial Engineering*, 110, 277–288.

Martens, A., and Vanhoucke, M., 2017, "A buffer control method for top-down project control", *European Journal of Operational Research*, 262(1), 274–286.

Kerkhove, L.-P. and Vanhoucke, M., 2017, "A parallel multi-objective parallel scatter search for optimising incentive contract design in projects", *European Journal of Operational Research*, 261(3), 1066–1084.

Kerkhove, L.-P. and Vanhoucke, M., 2017, "Optimised scheduling for weather sensitive offshore construction projects", *Omega - The International Journal of Management Science*, 66, 58–78.

Wauters, M., and Vanhoucke, M., 2017, "A nearest neighbour extension to project duration forecasting with artificial intelligence", *European Journal of Operational Research*, 259(3), 1097–1111.

Batselier, J. and Vanhoucke, M., 2017, "Improving project forecast accuracy by integrating earned value management with exponential smoothing and reference class forecasting", *International Journal of Project Management*, 35, 28–43.

Leyman, P. and Vanhoucke, M., 2017, "Capital- and resource-constrained project scheduling with net present value optimization", *European Journal of Operational Research*, 256(3), 757–776.

Kerkhove, L.-P. and Vanhoucke, M., 2017, "Extensions of earned value management: Using the earned incentive metric to improve signal quality", *International Journal of Project Management*, 35, 148–168.

Verbeeck, C., Van Peteghem, V., Vanhoucke, M., Vansteenwegen, P., and Aghezzaf, E.-H., 2017, "A metaheuristic solution approach for the time-constrained project scheduling problem", *OR Spectrum*, 39(2), 353–371.

Batselier, J. and Vanhoucke, M., 2017, "Project regularity: Development and evaluation of a new project characteristic", *Journal of Systems Science and Systems Engineering*, 26(1), 100–120.

Maenhout, B. and Vanhoucke, M., 2017, "A resource type analysis of the integrated project scheduling and personnel staffing problem", *Annals of Operations Research*, 252(2), 407–433.

Batselier, J. and Vanhoucke, M., 2016, "Practical application and empirical evaluation of reference class forecasting for project management", *Project Management Journal*, 47(5), 36–51.

Wauters, M. and Vanhoucke, M., 2016, "A study on complexity and uncertainty perception and solution strategies for the time/cost trade-off problem", *Project Management Journal*, 47(4), 29–50.

Leyman, P. and Vanhoucke, M., 2016, "Payment models and net present value optimization for resource-constrained project scheduling", *Computers and Industrial Engineering*, 91, 139–153.

Wauters, M. and Vanhoucke, M., 2016, "A comparative study of artificial Intelligence methods for project duration forecasting", *Expert Systems with Applications*, 46, 249–261.

Kerkhove, L.-P. and Vanhoucke, M., 2016, "Incentive contract design for projects: The owner's perspective", *Omega - The International Journal of Management Science*, 62, 93–114.

Maenhout, B. and Vanhoucke, M., 2016, "An exact algorithm for an integrated project staffing problem with a homogeneous workforce", *Journal of Scheduling*, 19(2), 107–133.

Vanhoucke, M. and Coelho, J., 2016, "An approach using SAT solvers for the RCPSp with logical constraints", *European Journal of Operational Research*, 249(2), 577–591.

- Vanhoucke, M. and Colin, J., 2016, "On the use of multivariate regression methods for longest path calculations from earned value management observations", *Omega - The International Journal of Management Science*, 61, 127–140.
- Colin, J. and Vanhoucke, M., 2016, "Empirical perspective on activity durations for project management simulation studies", *Journal of Construction Engineering and Management*, 142(1), 04015047.
- Vandenheede, L., Vanhoucke, M. and Maenhout, B., 2016, "A scatter search for the extended resource renting problem", *International Journal of Production Research*, 54(16), 4723–4743.
- Batselier, J. and Vanhoucke, M., 2015, "Empirical evaluation of earned value management forecasting accuracy for time and cost", *Journal of Construction Engineering and Management*, 141(11), 1–13.
- Cardoen, B., Beliën, J. and Vanhoucke, M., 2015, "On the design of custom packs: Grouping of medical disposable items for surgeries", *International Journal of Production Research*, 53(24), 7343–7359.
- Colin, J., Martens, A., Vanhoucke, M. and Wauters, M., 2015, "A multivariate approach to top down project control using earned value management", *Decision Support Systems*, 79, 65–76.
- Batselier, J. and Vanhoucke, M., 2015, "Construction and evaluation framework for a real-life project database", *International Journal of Project Management*, 33(3), 697–710.
- Batselier, J. and Vanhoucke, M., 2015, "Evaluation of deterministic state-of-the-art forecasting approaches for project duration based on earned value management", *International Journal of Project Management*, 33(7), 1588–1596.
- Van Peteghem, V. and Vanhoucke, M., 2015, "Influence of learning in resource-constrained project scheduling", *Computers and Industrial Engineering*, 87, 569–579.
- Colin, J. and Vanhoucke, M., 2015, "A comparison of the performance of various project control methods using earned value management systems", *Expert Systems with Applications*, 42(6), 3159–3175.
- Colin, J. and Vanhoucke, M., 2015, "Developing a framework for statistical process control approaches in project management", *International Journal of Project Management*, 33(6), 1289–1300.
- Van Huele, C. and Vanhoucke, M., 2015, "Operating theatre modelling: integrating social measures", *Journal of Simulation*, 9, 121–128.
- Leyman, P. and Vanhoucke, M., 2015, "A new scheduling technique for the resource-constrained project scheduling problem with discounted cash flows", *International Journal of Production Research*, 53(9), 2771–2786.
- Sels, V., Coelho, J., Dias, A.M. and Vanhoucke, M., 2015, "Hybrid tabu search and a truncated branch-and-bound for the unrelated parallel machine scheduling problem", *Computers and Operations Research*, 53, 107–117.
- Vanhoucke, M., 2015, "Controlling projects", *Journal of Modern Project Management*, 3(1), 5–5.
- Vanhoucke, M., 2015, "On the use of Schedule Risk Analysis for Project Management", *Journal of Modern Project Management*, 2(3), 108–117.
- Wauters, M. and Vanhoucke, M., 2015, "A study of the stability of Earned Value Management forecasting", *Journal of Construction Engineering and Management*, 141(4), 1–10.
- Willems, L. and Vanhoucke, M., 2015, "Classification of articles and journals on project control and earned value management", *International Journal of Project Management*, 33(7), 1610–1634.
- Colin, J. and Vanhoucke, M., 2014, "Setting tolerance limits for statistical project control Using Earned Value Management", *Omega - The International Journal of Management Science*, 49, 107–122.
- Van Huele, C. and Vanhoucke, M., 2014, "Analysis of the integration of the physician rostering problem and the surgery scheduling problem", *Journal of medical systems*, 38(6), 1–16.
- Van Huele, C. and Vanhoucke, M., 2014, "Decomposition-based heuristics for the integrated physician rostering and surgery scheduling problem", *Health Systems*, 1–17.
- Kerkhove, L.-P. and Vanhoucke, M., 2014, "Scheduling of unrelated parallel machines with limited server availability on multiple production locations: a case study in knitted fabrics", *International Journal of Production Research*, 52(9), 2630–2653.
- Van Peteghem, V. and Vanhoucke, M., 2014, "An experimental investigation of metaheuristics for the multi-mode resource-constrained project scheduling problem on new dataset instances", *European Journal of Operational Research*, 235(1), 62–72.
- Sels, V. and Vanhoucke, M., 2014, "A hybrid Electromagnetism-like Mechanism/tabu search procedure for the single machine scheduling problem with a maximum lateness objective", *Computers and Industrial Engineering*, 67, 44–55.

- Vanhoucke, M., 2014, "Blended learning in Project Management: An overview of the Operations Research & Scheduling group", *Journal of Modern Project Management*, 1(3), 108–121.
- Vanhoucke, M., 2014, "Praise youth and it will prosper: PMI Belgium's recognition of young PM Potential", *Journal of Modern Project Management*, 2(2), 112–117.
- Vanhoucke, M., 2014, "Teaching Integrated Project Management and Control: Enhancing student learning and engagement", *Journal of Modern Project Management*, 1(4), 99–107.
- Wauters, M. and Vanhoucke, M., 2014, "Support Vector Machine regression for project control forecasting", *Automation in Construction*, 47, 92–106.
- Maenhout, B. and Vanhoucke, M., 2013, "An integrated nurse staffing and scheduling analysis for longer-term nursing staff allocation problems", *Omega - The International Journal of Management Science*, 41(2), 485–499.
- Maenhout, B. and Vanhoucke, M., 2013, "Analyzing the nursing organizational structure and process from a scheduling perspective", *Health Care Management Science*, 16(3), 177–196.
- Maenhout, B. and Vanhoucke, M., 2013, "Reconstructing nurse schedules: computational insights in the problem size parameters", *Omega - The International Journal of Management Science*, 41(5), 903–918.
- Van Peteghem, V. and Vanhoucke, M., 2013, "An artificial immune system algorithm for the resource availability cost problem", *Flexible Services and Manufacturing Journal*, 25(1-2), 122–144.
- Vanhoucke, M., 2013, "An overview of recent research results and future research avenues using simulation studies in project management", *ISRN Computational Mathematics*, 1–19.
- Vanhoucke, M., 2013, "Project baseline scheduling: An overview of past experiences", *Journal of Modern Project Management*, 1(2), 18–27.
- Maenhout, B. and Vanhoucke, M., 2013, "An artificial immune system based approach for solving the nurse rostering system", in M. Middendorf & C. Blum, eds. *Lecture Notes in Computer Science*. Berlin Heidelberg: Springer-Verlag, pp. 97–108.
- Vanhoucke, M., 2013, "Integrated project controls: using operations research methods to improve the efficiency of project control", *Lecture Notes in Management Science*, 5, 174–175.
- Sels, V., Gheysen, N. and Vanhoucke, M., 2012, "A comparison of priority rules for the job shop scheduling problem under different flow time- and tardiness-related objective functions", *International Journal of Production Research*, 50(15), 4255–4270.
- Sels, V. and Vanhoucke, M., 2012, "A hybrid genetic algorithm for the single machine maximum lateness problem with release times and family setups", *Computers and Operations Research*, 39(10), 2346–2358.
- Vanhoucke, M., 2012, "Measuring the efficiency of project control using fictitious and empirical project data", *International Journal of Project Management*, 30(2), 252–263.
- Vanhoucke, M., 2012, "Operations research and dynamic project scheduling: When research meets practice", *Lecture Notes in Management Science*, 4, 1–8.
- Coelho, J. and Vanhoucke, M., 2011, "Multi-mode resource-constrained project scheduling using RCPSP and SAT solvers", *European Journal of Operational Research*, 213(1), 73–82.
- Maenhout, B. and Vanhoucke, M., 2011, "An evolutionary approach for the nurse rostering problem", *Computers and Operations Research*, 38(10), 1400–1411.
- Van Peteghem, V. and Vanhoucke, M., 2011, "Using resource scarceness characteristics to solve the multi-mode resource-constrained project scheduling problem", *Journal of Heuristics*, 17(6), 705–728.
- Sels, V., Craeymeersch, K. and Vanhoucke, M., 2011, "A hybrid single and dual population search procedure for the job shop scheduling problem", *European Journal of Operational Research*, 215(3), 512–523.
- Sels, V., Steen, F. and Vanhoucke, M., 2011, "Applying a hybrid job shop procedure to a Belgian manufacturing company producing industrial wheels and castors in rubber", *Computers and Industrial Engineering*, 61(3), 697–708.
- Sels, V. and Vanhoucke, M., 2011, "A hybrid dual-population genetic algorithm for the single machine maximum lateness problem", in P. Merz & J.-K. Hao, eds. *Lecture Notes in Computer Science*. Berlin, Germany: Springer-Verlag, pp. 14–25.
- Vanhoucke, M., 2011, "On the dynamic use of project performance and schedule risk information during project tracking", *Omega - The International Journal of Management Science*, 39(4), 416–426.
- Maenhout, B. and Vanhoucke, M., 2010, "A hybrid scatter search heuristic for personalized crew rostering in the airline industry.", *European Journal of Operational Research*, 206(1), 155–167.

- Maenhout, B. and Vanhoucke, M., 2010, "Branching strategies in a branch-and-price approach for a multiple objective nurse scheduling problem", *Journal of Scheduling*, 13(1), 77–93.
- Van Peteghem, V. and Vanhoucke, M., 2010, "A genetic algorithm for the preemptive and non-preemptive multi-mode resource-constrained project scheduling problem", *European Journal of Operational Research*, 201(2), 409–418.
- Vanhoucke, M., 2010, "A scatter search heuristic for maximising the net present value of a resource-constrained project with fixed activity cash flows", *International Journal of Production Research*, 48(7), 1983–2001.
- Vanhoucke, M., 2010, "Introducing Optimization Techniques to Students: An Exam Case Distribution Model", *INFORMS Transactions on Education*, 10(2), 53–61.
- Vanhoucke, M., 2010, "Using activity sensitivity and network topology information to monitor project time performance.", *Omega - The International Journal of Management Science*, 38(5), 359–370.
- Maenhout, B. and Vanhoucke, M., 2009, "The impact of incorporating nurse-specific characteristics in a cyclical scheduling approach", *Journal of the Operational Research Society*, 60(12), 1683–1698.
- Vanhoucke, M. and Debels, D., 2009, "A finite-capacity production scheduling procedure for a Belgian steel company", *International Journal of Production Research*, 47(3), 561–584.
- Vanhoucke, M. and Maenhout, B., 2009, "On the characterization and generation of nurse scheduling problem instances", *European Journal of Operational Research*, 196(2), 457–467.
- Van Peteghem, V. and Vanhoucke, M., 2009, "An artificial immune system for the multi-mode resource-constrained project scheduling problem", *Lecture Notes in Computer Science*, 5482, 85–96.
- Vanhoucke, M., 2009, "A genetic algorithm for net present value maximization for resource constrained projects", in C. Cotta & P. Crowling, eds. *Lecture Notes in Computer Science*. Berlin, Germany: Springer Verlag, pp. 13–24.
- Labro, E. and Vanhoucke, M., 2008, "Diversity in resource consumption patterns and robustness of costing systems to errors", *Management Science*, 54(10), 1715–1730.
- Maenhout, B. and Vanhoucke, M., 2008, "Comparison and hybridization of crossover operators for the nurse scheduling problem", *Annals of Operations Research*, 159(1), 333–353.
- Vanhoucke, M., Coelho, J., Debels, D., Maenhout, B. and Tavares, L.V., 2008, "An evaluation of the adequacy of project network generators with systematically sampled networks", *European Journal of Operational Research*, 187(2), 511–524.
- Vanhoucke, M., 2008, "Setup times and fast tracking in resource-constrained project scheduling", *Computers and Industrial Engineering*, 54(4), 1062–1070.
- Vanhoucke, M. and Debels, D., 2008, "The impact of various activity assumptions on the lead time and resource utilization of resource-constrained projects", *Computers and Industrial Engineering*, 54(1), 140–154.
- Debels, D. and Vanhoucke, M., 2007, "A decomposition-based genetic algorithm for the resource-constrained project-scheduling problem", *Operations Research*, 55(3), 457–469.
- Labro, E. and Vanhoucke, M., 2007, "A simulation analysis of interactions among errors in costing systems", *The Accounting Review*, 82(4), 939–962.
- Maenhout, B. and Vanhoucke, M., 2007, "An electromagnetic meta-heuristic for the nurse scheduling problem", *Journal of Heuristics*, 13(4), 359–385.
- Maenhout, B. and Vanhoucke, M., 2007, "NSPLib – A Nurse Scheduling Problem Library: a tool to evaluate (meta-)heuristic procedures", in S. Brailsford & P. Harper, eds. *Operational Research for Health Policy: Making Better Decisions*. Amsterdam: Elsevier, pp. 151–165.
- Vanhoucke, M., 2007, "Work continuity optimization for the Westerscheldetunnel project in the Netherlands", *Tijdschrift voor Economie en Management*, 52, 435–449.
- Vanhoucke, M. and Debels, D., 2007, "The discrete time/cost trade-off problem: extensions and heuristic procedures", *Journal of Scheduling*, 10(4-5), 311–326.
- Vanhoucke, M. and Vandevoorde, S., 2007, "A simulation and evaluation of earned value metrics to forecast the project duration", *Journal of the Operational Research Society*, 58, 1361–1374.
- Debels, D., De Reyck, B., Leus, R. and Vanhoucke, M., 2006, "A hybrid scatter search/electromagnetism meta-heuristic for project scheduling", *European Journal of Operational Research*, 169(2), 638–653.
- Debels, D. and Vanhoucke, M., 2006, "The electromagnetism meta-heuristic applied to the resource-constrained project scheduling problem", *Lecture Notes in Computer Science*, 3871, 259–270.

Maenhout, B. and Vanhoucke, M., 2006, "New computational results for the nurse scheduling problem: A scatter search algorithm", *Lecture Notes in Computer Science*, 3906, 159–170.

Vandevoorde, S. and Vanhoucke, M., 2006, "A comparison of different project duration forecasting methods using earned value metrics", *International Journal of Project Management*, 24, 289–302.

Vanhoucke, M., 2006, "An efficient hybrid search algorithm for various optimization problems", *Lecture Notes in Computer Science*, 3906, 272–283.

Vanhoucke, M., 2006, "Scheduling an R&D project with quality-dependent time slots", *Lecture Notes in Computer Science*, 3982, 621–630.

Vanhoucke, M., 2006, "Work continuity constraints in project scheduling", *Journal of Construction Engineering and Management*, 132(1), 14–25.

Debels, D. and Vanhoucke, M., 2005, "A bi-population based genetic algorithm for the resource-constrained project scheduling problem", *Lecture Notes in Computer Science*, 3483, 378–387.

Vanhoucke, M., 2005, "New computational results for the discrete time/cost trade-off problem with time-switch constraints", *European Journal of Operational Research*, 165(2), 359–374.

Vanhoucke, M., Vereecke, A. and Gemmel, P., 2005, "The project scheduling game (PSG): simulating time/cost trade-offs in projects", *Project Management Journal*, 36(1), 51–59.

Demeulemeester, E., Vanhoucke, M. and Herroelen, W., 2003, "RanGen: A random network generator for activity-on-the-node networks", *Journal of Scheduling*, 6(1), 17–38.

Vanhoucke, M. and Demeulemeester, E., 2003, "The application of project scheduling techniques in a real-life environment", *Project Management Journal*, 34, 30–42.

Vanhoucke, M., Demeulemeester, E. and Herroelen, W., 2003, "Progress payments in project scheduling problems", *European Journal of Operational Research*, 148(3), 604–620.

Vanhoucke, M., Demeulemeester, E. and Herroelen, W., 2002, "Discrete time/cost trade-offs in project scheduling with time-switch constraints", *Journal of the Operational Research Society*, 53(7), 741–751.

Vanhoucke, M., Demeulemeester, E. and Herroelen, W., 2001a, "An exact procedure for the resource-constrained weighted earliness-tardiness project scheduling problem", *Annals of Operations Research*, 102, 179–196.

Vanhoucke, M., Demeulemeester, E. and Herroelen, W., 2001b, "Maximizing the net present value of a project with linear time-dependent cash flows", *International Journal of Production Research*, 39(14), 3159–3181.

Vanhoucke, M., Demeulemeester, E. and Herroelen, W., 2001c, "On maximizing the net present value of a project under renewable resource constraints", *Management Science*, 47(8), 1113–1121.

Demeulemeester, E., De Reyck, B., Foubert, B., Herroelen, W. and Vanhoucke, M., 1998, "New computational results on the discrete time/cost trade-off problem in project networks", *Journal of the Operational Research Society*, 49(11), 1153–1163.

### Non-peer reviewed international articles

André, P., Vanhoucke, M. and Floriano Salvaterra, 2015, "introdução à duração agregada", *Mundo Project Management*, 62, 56–65.

Vanhoucke, M., 2014, "Gerenciamento Integrado de Controles De Projetos: Primeiro vem a teoria, e então a prática", *Mundo Project Management*, 59, 52–56.

Vanhoucke, M., 2013a, "Measuring schedule adherence", *The Measurable News*, 4, 21–26.

Sheibani, K., Montemanni, R., Nordlander, T., Hirsch, P. and Vanhoucke, M., 2014, "Proceedings of the 6th International Conference of Applied Operational Research.", in *Lecture Notes in Management Science*. p. 246.

Vanhoucke, M., 2013, "Project Management using dynamic scheduling: Baseline scheduling, risk analysis and project control", *The Measurable News*, 2, 45–50.

Vanhoucke, M., 2012, "Dynamic scheduling: Integrating schedule risk analysis with earned value management", *The Measurable News*, (2), 11–13.

Vanhoucke, M. and Shtub, A., 2011, "Adding value to earned value analysis", *PM World Today*, 13(1).

Vanhoucke, M., 2010, "Measuring Time: An earned value performance management study", *The Measurable News*, 1, 10–14.

Vanhoucke, M. and Vandevoorde, S., 2009, "Forecasting a project's duration under various topological structures", *The Measurable News*, (Spring), 26–30.

Vanhoucke, M. and Vandevoorde, S., 2008, "Earned value forecast accuracy and activity criticality", *The Measurable News*, (Summer), 13–16.

Vanhoucke, M., 2007, "De studie naar kwaliteit: een levenslange garantie", *Management Jaarboek*, 51–53.

Vanhoucke, M. and Vandevoorde, S., 2007, "Measuring the accuracy of earned value/earned schedule forecasting predictors", *The Measurable News*, Winter, 26–30.

Vanhoucke, M., 2006, "Waarschuwingssystemen: veel te vroeg of net te laat?", *Management Jaarboek*, 121–124.

Vanhoucke, M. and Vandevoorde, S., 2006, "Waar zit het tijdsaspect in een 'earned value'-meetsysteem?", *Management Jaarboek*, 131–137.

Vanhoucke, M. and Deschoolmeester, D., 2005, "Supply chain management: waar de klant nog steeds koning is", *Management Jaarboek*, 113–115.

Vanhoucke, M., 2003, "Projectplanning – niet alleen tijd is belangrijk", *Management Jaarboek*, 118–124.

### Book chapters

Coelho, J. and Vanhoucke, M., 2015, "The Multi-Mode Resource-Constrained Project Scheduling Problem", in C. Schwindt & J. Zimmermann, eds. *Handbook on Project Management and Scheduling Vol.1*. Springer, pp. 491–511.

Van Peteghem, V. and Vanhoucke, M., 2015, "Heuristic Methods for the Resource Availability Cost Problem", in C. Schwindt & J. Zimmermann, eds. *Handbook on Project Management and Scheduling Vol.1*. Springer, pp. 339–359.

Vanhoucke, M., 2015, "Generalized Discrete Time-Cost Tradeoff Problems", in C. Schwindt & J. Zimmermann, eds. *Handbook on Project Management and Scheduling Vol.1*. Springer, pp. 639–658.

Vanhoucke, M., 2014, "Managing cost and earned value", in R. Turner, ed. *Gower Handbook of Project Management*. Gower Publishing, pp. 247–261.

Vanhoucke, M., 2013, "The impact of project schedule adherence and rework on the duration forecast accuracy of earned value metrics", in E. C. Hoffmann, ed. *Project Management: Practices, Challenges and Developments*. Nova Science Publishers, pp. 95–131.

Vanhoucke, M. and Maenhout, B., 2013, "An empirical investigation of different solution strategies for meta-heuristic optimization: Solution representation, diversity and space reduction", in P. Siarry, ed. *Heuristics: Theory and Applications*. Nova Science Publishers, pp. 53–68.

Sels, V. and Vanhoucke, M., 2012, "Genetic algorithms for single machine scheduling problems: a trade-off between intensifications and diversification", in A. R. Muñoz & I. G.

Rodriguez, eds. *Handbook of Genetic Algorithms: New Research*. Nova Science Publishers, pp. 265–293.

Maenhout, B. and Vanhoucke, M., 2011, "Days on and days off scheduling of pilots under a variable workload", in C. R. Walsh, ed. *Airline industry: Strategies, Operations and Safety*. Nova Science Publishers, pp. 193–212.

Vanhoucke, M., 2009, "Static and dynamic determinants of earned value based time forecast accuracy", in T. T. Kidd, ed. *Handbook of Research on Technology Management's Planning and Operations*. IGI Global, pp. 361–374.

### Books

Vanhoucke, M., 2015, "Project management knowledge center: The technical guide for baseline scheduling, schedule risk analysis and project control" 1st ed., Springer (in press).

Vanhoucke, M., 2017, "Taking sound business decisions: From rich data to better solutions" 2nd ed., OR-AS: 94 pages.

Vanhoucke, M., 2017, "The art of project management: A story about work and passion" 4th ed., OR-AS: 93 pages.

Vanhoucke, M., 2014, "Integrated project management and control: First comes the theory, then the practice" 1st ed., Springer: 141 pages.

Vanhoucke, M., 2013, "Project management with dynamic scheduling: Baseline scheduling, risk analysis and project control" 2nd ed., Springer: 318 pages.

Vanhoucke, M., 2010, "Dynamic scheduling on your desktop" 1st ed., OR-AS: 63 pages.

Vanhoucke, M., 2010, "Measuring time: Improving project performance using earned value management" 1st ed., Springer: 164 pages.

# Epilogue

“Alone we can do so little. Together we can do so much.” - Helen Keller

## Epilogue

### Credits

Text written by Mario Vanhoucke  
Cover downloaded from [www.pixabay.com](http://www.pixabay.com)  
Layouting done by Mario Vanhoucke, Gaëtane Beernaert and Sokko  
Editing and proofreading done by Gaëtane Beernaert

### Thanks

A special thank you goes to all the people I have met throughout the years who were, and still are a source of inspiration.

While the quantitative methods in decision making can be applied everywhere, my first and primary use lies in the field of Project Management and Scheduling. A special word of thanks goes to the many people working in this field of Project Management (people from **PMI Belgium**, **EVM Europe** and the **College of Performance Management**) with an interest in quantitative decision making for managing projects.

Much of the methods discussed in this book come from various meetings with members from the **EURO Working Group on Project Management and Scheduling** community. Ever since the start of my PhD in 1996, I have met people across the world with an interest in optimisation models and algorithms and data analysis techniques. My recent membership to the board of this wonderful organisation is something I consider as very valuable and honourable.

A great word of thanks goes to the people I have met at my work at **Ghent University**, **Vlerick Business School**, **UCL School of Management** of **University College London**, **Peking University** and **OR-AS**. The inspiration I received from (MBA) students from America, Europe and Asia has been, and always will be, an unforgettable learning lesson that I use everywhere, in my teaching activities, in my research, and in my consultancy. Thank you.

The meetings with decision makers (mainly in Belgium, Portugal and the UK) have forced me to put the theoretical concepts into the right perspective. They showed me that these methods can be used ... for real!

Last but not least, the intense work with my team at the **OR&S** research group of Ghent University has brought us where we are right now, and will probably bring us to new directions in the coming years. It's a wonderful team!

I won't mention names. Just a thank you to all of you.

### Update

This book will be updated on a regular basis to add the new topics and themes we will investigate. If you want to receive a message when an update is available, connect through @ORASTalks.

First edition: September - 2015  
Second edition: September - 2017

© 2017 OR-AS

Operations Research - Applications and Solutions

[www.or-as.be](http://www.or-as.be)

